Plays, Values, Analysis and the Complexity of Chinese Chess

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1 Introduction and Motivation

Chinese Chess (Xiangqi) and Backgammon evolved from an ancient Chinese game called Liubo that was invented some 3,500 years ago [10, pp. 3–11]. Like Liubo, Xiangqi consists of one king, five pawns, and several higher order pieces, and the game is decided by capturing the opponent's king. Unlike in Liubo, the moves of Xiangqi are not determined by the roll of dice. The dice part of the game eventually evolved into another famous game, Backgammon. Xiangqi predates both Shogi and International Chess, and the latter are the obvious variants of the former. In this paper we will refer to International Chess as Chess and Chinese Chess as Xiangqi. Since Xiangqi and Chess are both purely strategic games (without random moves), it should conceptually be possible to decide whether there is a winning strategy for the first or the second player, or the game always ends with a draw, if both the players play perfect strategies. Such games are called combinatorial games, and combinatorial game theory (CGT) is the branch of mathematics devoted to their analysis [3]. In recent times a rich theory on how to evaluate/analyze game positions has been developed. This theory has successfully been applied to certain endgame positions of Chess [6] and Go [2].

Unfortunately, CGT cannot be directly applied to Xiangqi. Therefore in this paper, we will make a few simplifying assumptions about the game to make it better accessible to CGT. These assumptions will of course affect the actual value of a game position but we will still be able to correctly identify the winner, which is the foremost important part in analysis of combinatorial board games.

In Chess, pawn endgames can be easily analyzed if there are no higher order pieces present on the board [6]. By higher order pieces we mean all pieces except pawns, e.g. Bishop, Rook, etc. In Xiangqi this is not an easy job, since pure pawn endgames are rare because higher order pieces usually survive until the end, leading to loopy games with more complex values [3]. Another problem in strategic games is to find a winning move for a given game position. Since the search space usually becomes astronomically huge with every move, finding the best move by exhaustive search is not a good idea at all. Actually, most interesting games are PSPACE- or

EXPTIME-complete. There exist many endgame rules forbidding the perpetual chase of some opponent's piece [9], and experience with other board games shows that such rules usually make a game polynomially intractable.

The rest of the paper is categorized as follows. In Section 2 we make some justifiable assumptions on the movements of the pieces to make Xiangqi better accessible to CGT. Section 3 will comprise of some initial discussion on identifying game values in Xiangqi. In Section 4 we extend the analysis to end games with complex values. In particular, we construct game positions of value *over*, *tis* and *tisn*, *on* and *off*, and *dud*. We will finally end with some useful discussion and propose some open problems.

2 Preliminaries

We will start off by briefly reviewing CGT. This is important for readers who have little knowledge about CGT. In the latter half, we will propose a few restrictions to piece movements in Xiangqi to make the game better suited for analysis by CGT.

Combinatorial Games: A combinatorial game is a two-player perfect information game without chance moves [3, 4, 8]. The number of positions is usually finite, and there is a particular starting position. The players are usually called Left and Right. In Xiangqi, we use Red for Left, and Black for Right. The players move alternately according to clearly defined rules, usually with Left making the first move. The positions that can be reached by some move of Left (Right) are called Left's (Right's) options. A game can be represented by its game graph, where nodes are the game positions and directed arcs are the options [3]. A play is a path in this graph from the starting node to some terminal node, or an infinite path which represents a draw. A game which can result in a draw is called a loopy game. In a normal play, the first player unable to move loses the play and his opponent wins. In misére play the first player unable to move wins. We say a player wins a game if he has a winning strategy, i.e., he can force the play based on some strategy to end in a winning position. A draw is a position from which neither player can force a win, and each player has a non-losing next move. Some games, e.g. Chess can also end in a tie, i.e., positions where the play ends but neither player wins. Typical examples of combinatorial board games are Chess, NineMen Morris, Checkers, and Go. Games where the players do not have the same moves available from the same game position are called partizan games, otherwise they are impartial games. Note that in the game graph of partizan games we must distinguish between edges corresponding to Left's and Right's options. In impartial games, each player can use any edge of the game graph. A combinatorial game without draws or ties can have four different outcomes if we assume that both players play perfect game: Left/Right player wins (whether he starts or not), or first/second player to move wins. The ultimate goal of CGT is to classify all combinatorial games into these four categories. Even if it is known which player wins, it can be very difficult to find a winning move. It is known that in Hex the first player has a winning strategy but this strategy still has to be discovered [4]. To better understand the structure of games Conway introduced the surreal numbers [3, 4] by generalizing the real and ordinal number system. They are given by a simple recursive definition [4, pp. 4]: *If L and R are any two sets of numbers, and no member of L is $\geq$ any member of R, then there is a number*

$\{L|R\}$. *All numbers are constructed in this way.* For more information and some recent results readers are encouraged to see [5].

In Xiangqi we will always call the two players Red (White in Chess) and Black instead of Left and Right, respectively. All games will be seen from Red's perspective. We will not use the traditional way of writing down Xiangqi moves for the simple reason that the conventions used are hard to decipher. Instead, we will use the more intuitive Chess notation popularly known as algebraic notation. It is important to note that contrary to Chess in Xiangqi the rows of the board are numbered from top to bottom. When we try to apply CGT to analyze Xiangqi positions we face several difficulties. CGT assumes that a game always has a winner, i.e. there is no room for draws. Therefore, Chess and its variants are strictly speaking not combinatorial games. But Moews [11] showed that it is possible to extend CGT to Go, also a loopy game. Since in Xiangqi no player seems to have a real advantage from choosing repeating moves we restrict them upfront.

Assumption 1: The King piece is not allowed to move.

With this assumption, the position in Figure 1 has value $\star$ (see Section 3), i.e., it is a first player win. But without the assumption the second player's best option would be to perpetually move its king, eventually forcing the first player to do the same, resulting in a draw. Berlekamp et al. [3, pp. 317] call this situation a *dud*. Readers who do not follow this argument will do perfectly well by ignoring it for the time being. Detailed analysis will be reviewed in Section 3.

Assumption 1 is actually equivalent to say that the subgame played by the two kings on their respective 3×3 roaming areas has value zero. Since the kings could move forever, resulting in a draw, it simply kills the purpose of analysis. But it is close enough since we could augment CGT to include draws by arbitrarily treating a draw as a second player win. We can support this argument by saying that at least, it is a position where the first player cannot win. We put a similar restriction on pawns. Since pawns can move horizontally after crossing the river in the middle of the board they are another potential source of loopy games.

Assumption 2: The pawns after they cross the river can only move horizontally if there is a possibility to capture an opponent's piece.

In particular, after the opponent has lost all his pieces the pawns can only move forward. This allows the pawns to move horizontally, but only in one direction until they hit the board boundary. With this assumption we will usually underestimate the value of a game position because we deprive a player of additional spare moves, but we will still be able to predict the right winner. Elkies [6] observed that another drawback of Chess is the rather small board size, compared to the Go board. In particular, most pieces can have a far reaching impact. This is also true for Xiangqi. As a consequence, it is very difficult to subdivide a game into several independent subgames, a key step in CGT analysis. Although the Xiangqi board is slightly bigger (9×10) than the Chess board (8×8) it has only five pawns besides the pairs of higher order pieces (rook, guardian, cannon, and elephant). The rules of Xiangqi [9] are more complex than in Chess and players rely more on higher order pieces rather than pawns.

Assumption 3: Subgames on different parts of the board independently of each other.

We usually consider subgames on a single file or on a collection of files. We denote by $val(x)$

the value of the subgame on file x, and by $val(x-y)$ the subgame on files $x, x+1, \ldots, y$. The assumption that pieces in one subgame cannot interact with the pieces in another subgame is clearly a strong restriction, and it might easily result in a wrong evaluation of a game position. In Chess pawn endgames, a subgame usually ends with the pawns blocking each other. This is not possible in Xiangqi. Therefore, we only analyze subgames until the first piece is captured.

Assumption 4: A subgame ends either when some piece is captured or when there exists no piece that can be moved.

Conceptually, after a piece has been captured all other pieces are considered immobile, so we consider this to be a subgame of value 0. In Section 4 we will discuss some implications of Assumption 4. We note that Elkies implicitly introduced a similar assumption when he analyzed Chess positions [6]. He lets a game end when one player could be forced to move a piece in a trébuchet (mutual Zugzwang) position, thus eventually losing the game.

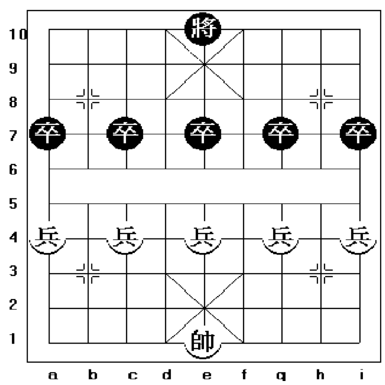

Figure 1: Game value $\star$.

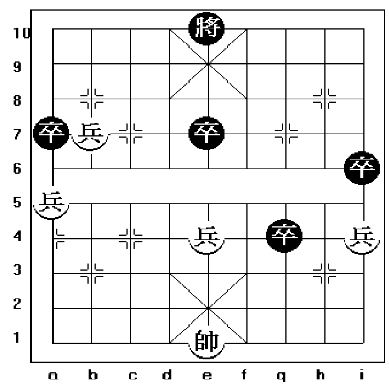

Figure 2: Game value $\{\uparrow|0,\star\}$.

3 A Taste of Analysis

In this section we will briefly identify some game positions. We deliberately shorten our identification of game positions to facilitate the readers and keep them interested. In Figure 1, the pawns and kings are at their initial positions. File `a` is win for the first player because $val(a) = \{0|0\} = \star$. Analogously, $val(c) = val(e) = val(g) = val(h) = \star$. The full board in Figure 1 has value $\star + \star + \star + \star + \star = \star$, pertaining the game to be a win for the first player to move. In Figure 2, we first consider the subgame of files `a` and `b`. Red has two options, 1. `a5a6` and 1. `b7xa7`. The former move will lead to a position of value $\star$, and the latter move will lead to a position of value 0. On the other hand, Black has only one option, 1. ... `a7a6`, leading to a position of value $\star$. Thus, $val(a-b) = \{0,\star|\star\}$. Since Red's option of $\star$ provides no real advantage, thus it is a reversible move [4, pp. 110]. Reversible moves can always be bypassed [3, pp. 62–66]. Therefore we arrive at values for files `a-b`: $val(a-b) = \{0|\star=\uparrow\}$. The subgame on files `a-b` is a winning position for Red, although with infinitesimally small advantage, i.e., Red does not have a full spare move.

We will now relax Assumption 4 to make readers aware of its implications. Red's pawn on `b7` could now move to the left to capture Black's pawn on `a7`, but afterwards can only move forward on the `a` file with aggregate of 3 extra moves. Also, if Black would first move its pawn to `a6` then the Red pawn could only move forward on the `b` file and never enter the `a` file. Thus, we would obtain $val(a-b) = \{7|\{7|-1\}\} = 7 + +_8$, a value only infinitesimally larger than 7. So the difference between the two game values, with or without Assumption 4, is approximately the seven extra moves that Red gains after it captures Black's. Even all these extra moves happen, surprisingly, the difference is not exactly 7. File `e` has value $val(e) = \star$. Thus, $val(a-b,e) =\uparrow \star$. Indeed, the two subgames on the two adjacent files `a-b` and `e` can be considered to be independent because Red's pawn on `b7` cannot reach Black's pawn on `e7` before it moves forward to `b6`. As we have seen before, file `i` alone would have value $val(i) = 0$. But we must also consider Black's pawn on `g4`. Thus, we consider the subgame consisting of the files `g` and `h`. Red has only one option, 1. `i4i5`, leading to a position of value $\star$. On the other hand, Black has two options, 1. ... `i6i5` and 1. ... `g4h4`. The former move also leads to a position of value $\star$. The latter move leads to a position symmetrical to the position on the `a-b` file, so it has value $val(a-b) =\downarrow$. Thus, $val(g-i) = \{\star|\star,\downarrow\} = \{\star|\downarrow\} =\downarrow 2$. So this subgame is a winning position for Black. But what is the value of the full board? Intuitively, we would expect a small advantage for Black because the position is nearly symmetrical, except for an additional spare move for Black with the pawn on `g4`. Indeed, we have $val(a-b, g-i) = \{\uparrow \star|0\}$, an infinitesimally small negative value. For the value of the full board we obtain $\uparrow + \star + \downarrow 2$ which is fuzzy, i.e., the board is a first player win. Red could start with 1. `i4i5`, leaving Black only the options 1. ... `i6xi5` (where Red would win with 2. e4e5), 1. ... `e7e6` (where Red would win with 2. `i5xi6`; note that this move would end the subgame on files `gi`), and 1. ... `a7a6` (where Red would win with 2. `i5xi6`). And similarly, Black could win by first moving 1. ... `a7a6`.

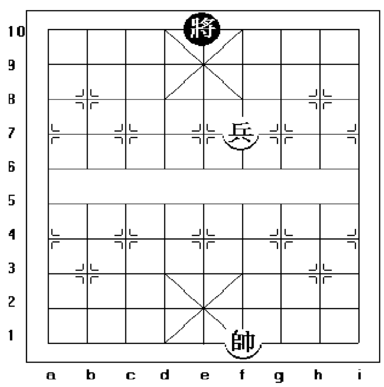

Figure 3: King and pawn vs. king.

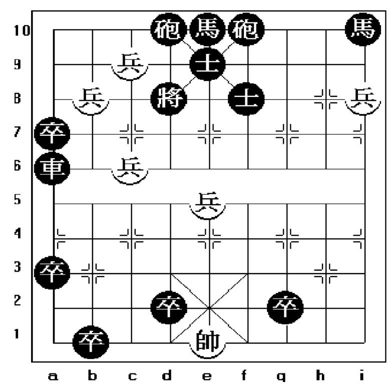

Figure 4: A classic endgame.

4 End Games

To analyze pure pawn endgames we introduced restrictions on the pawn movements (Assumption 2) to prevent loopy games. For higher order pieces we cannot easily do the same trick. Also, Assumption 4 does not seem to be useful anymore, since we want to be able to capture and capture back. In this section, we will introduce some more game values with examples from real end games. We will evidently find out that indeed CGT has limitations in analyzing complex games such as Xiangqi.

King Plus Pawn Against King: An important endgame position is Red with king and one pawn against Black who has only king (see Figure 3). This position is a win for Red. A winning move sequence is, for example, 1. `f7-f8 Ke10-e9`, 2. `Kf1-f2 Ke9-e10`, 3. `f8-f9 Ke10-d10`, 4. `f9-e9`, ... stalemate. What is this game's value? Red could straightforward attack the Black king, but it could also delay the attack by arbitrary king or pawn moves. Black can only sit there and wait for Red's attack. This is an example of a game called *over* [3, pp. 321]. Adding a Black pawn in Figure 3 would make the game infinite as neither of the opponents could force a win. Such a game is called a *dud* [3, pp. 317].

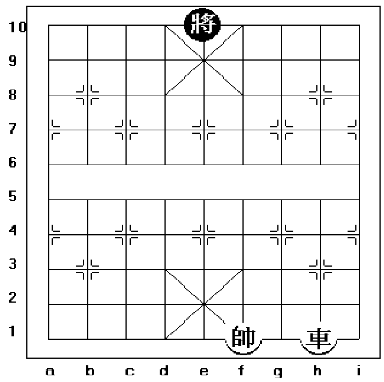

Figure 5: King and rook vs. king.

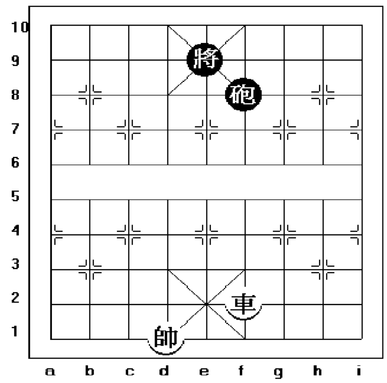

Figure 6: Tis and tisn.

A Classic Endgame: Analyzing endgames can be quite tricky if higher order pieces are involved. Figure 4 shows a classical endgame [9, pp. 77]. Red seems to be overpowered and outnumbered by Black. We cannot apply CGT to analyze this game. Remarkably, Red has a winning strategy if he can move first. All he has to do is to threaten the Black's king: 1. `b8-c8 Kd8-e8`, 2. `e5-e6 Ge9-d8` (to avoid a mate from 3. `e6-e7`), 3. `c9-d9 Gf8-e9` (to avoid another mate), 4. `e6-e7 Ke8-f8`, 5. `e7-e8 Kf8-f9` (in order not to expose the king directly), 6. `e8-e9 Gd8-e9`, 7. `d9-e8 Kf9-f8`, 8. `c8-d8 Ni10-g9` (to avoid another mate from `d8-c8`, Black moves his knight), 9. `i8-h8` (another threat on g8) `d2-d1`, 10. `Ke1-e2 g2-f3`, 11. `Ke2-e3 Cf10-g10`, 12. `d8-e8 Ng9-e8`, 13. `h8-g8`, ... and finally Red traps Black's king.

King Plus Rook Against King: Figure 5 shows a game where Red has the king and a rook, while Black only has the king. Red can win this game by occupying the central file `e` with the rook and the king, followed by checkmating the Black king either on file `d` or `f`. But again, Red can delay the winning moves, so this game has also value over.

Tis and Tisn: Consider the following game. Red has a pawn on `h6`, and Black has a pawn on `g7`. Red does not want not move `h6-h7` because then the Black pawn could move forward and the game would end as a draw. Red does also not want to move `h6-g6` because it would lose immediately. Similarly, Black does not want to move `g7-g6`. This is an example of a game called *tis* and *tisn* [3, pp. 322]. If Black moves a pawn first he loses the game. And if Red moves a pawn first he either loses the game or it ends in a draw. Thus, both sides always move their kings, and we have again an infinite game. If the Black pawn started at `g8` instead of `g7` then the situation would be better for Red. Red's first move 1. `h6-g6` would result in a subgame of value $val(g-h) = 0$ (which is better than *tis*), while Black's only option 1. ... `g8-g7` would lead to the previous game. We therefore write `tis=1+tisn`. Figure 6 is another example of `tis` and `tisn`. Assume Red just moved the rook to `f2` to threaten the Black cannon. Black's best answer is 1. ... `Ke9-f9`. If Red captures the rook the game will end as a draw: 2. `Rf2-f8 Kf9-f8`, and the two Kings have infinite moves.

On and Off: The simplest loopy games are *on* and *off* [3, pp. 316–320]. These are two independent subgames, where in *on* Red can move forever undisturbed by Black's moves, whereas in *off* Black can move forever undisturbed by Red's moves. These games could of course be constructed with pawns that move horizontally forever after crossing the river. But we can also employ guardians to get these positions. Their movement is restricted to the castle so they cannot interact with the opponent's king or guardians. But they can move as often as they like. Thus, a subgame where Red (Black) has one guardian and the king is an *on* (*off*). And the sum of these two games is a *dud*, i.e., an infinite game where no player can win or lose (a *tie*).

5 Summary

In this paper we shown a glimpse of the complex Xiangqi endgames. It is evident that the existing CGT is not really enough to perfectly and with easy analyze such endgames. This work is still in it preliminary stages. We hope that in future we would have a more solid and concrete theory for analyzing non–separable games like Xiangqi.

6 Conjectures and Open Problems

Our analysis are strongly based on the four assumptions. Without them, the endgames become hard and in some cases impossible to analyze. We pose here some open problems from our analysis of Xiangqi and understanding of CGT.

Conjecture 1: Xiangqi is EXPTIME–complete.

Our conjecture is based on similar results for other variants of Chess, e.g. Shogi [1]. We strongly believe that Xiangqi will lie in the EXPTIME– class if not, than at least it will be PSPACE–complete.

Conjecture 2: Xiangqi endgames without the four assumptions are at least NP–hard.

Often in board games, we are interested in analyzing the endgames. This is a natural phenomenon since the complexity of games reduces considerably when games converge towards their final stages. We based our second conjecture on similar lines as the first one, since in certain case Go endgames are PSPACE–hard [12]. Moreover, the classic game shown in this paper is a convincing example of the intractability of Xiangqi endgames.

We also seek a concrete and accessible theory (extention of CGT) which will enable researchers to analyze endgames of complex values.

7 Acknowledgements

The author would like to acknowledge the valuable suggestions and support offered by Rudolf Fleischer with whom the author wrote an initial and extended version of this paper [7].

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