COMBINATORIAL GAMES: AYO

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Abstract. Ayo is a Nigerian game played on a wooden board with stone pebbles. The game dates back to the ancient Egyptian Empire. An initial glance over the game suggests that its analysis would be very simple, yet it turns out that it is much more involved than suspected. On the other hand Ayo endgames are much easier to analyze and reveal interesting positions. In this paper we will introduce Ayo and the known results about it. Finally we will conclude with asking some open problems.

Introduction. There are various kinds of Mancala games dated as back as the early years of the great Egyptian Empire. Mancala games mostly played in Nigeria are a group of games, with certain common characteristics. They all involve cup-shaded depression called *pits* filled with stones or seeds. Players take turns and maneuver the stones, by various rules, which govern them. Ayo is one such two-player game Mancala game played in the western part of Nigeria. With predefined rules, Ayo players follow their respective strategy and do not depend on chance moves (dice). Thus Ayo, can be termed as a *combinatorial game* [Berlekamp et al. 1982].

What is Ayo? Ayoyayo (Ayo) is played over a wooden board 20 inches long, 8 inches wide, and 2 inches thick. This board accommodates two rows of six pits each 3 inches in diameter. The pits are filled with either stones or dried palm nuts [Broline and Loeb 1995].

The Play. Ayo is played with 48 stones with 4 stones placed in each of the 12 pits. Two players alternatively move the stones, each controlling 6 pits. Their objective is to capture as many stones of their opponents as possible. A move consists of a player choosing a non-empty pit on his side of the board and removes all of its stones. The stones are redistributed (*sown*) one stone per pit.

The pits are chosen in counterclockwise direction from the pit that has been chosen initially. A pit containing 12 or more stones is called an *Odu*. If the chosen pit is an Odu, the same redistribution continues, but the initial pit is skipped on each circuit of the board [Odeleya 1977]. A capture is made, when the last pit *sown* is on the opponent's side, and contains after the addition of the sowing stone either two or three stones. Thus the stones in the pit are captured and removed from the game. Also are captured the immediately preceding pits which meet the same conditions. One important feature of the game is that a player has to make a move such that his opponent has a legal move to play. If this does not happen, than the opponent is awarded with all the remaining stones on the board. If during the game, it is found that there are not enough stones to make a capture, but both players will always get a legal move, the game is stopped and the players are awarded stones that reside on their respective side of the board.

The initial game is more rapid and interesting, where both the players try to capture stones as quickly as possible. To determine the optimal strategy during the initial play is much harder than the endgame and thus has yet not been studied. It involves planning at least 2-3 moves in advance, and remembering the number of stones in every pit [Broline and Loeb 1995].

Known Results. The only referenced work at the moment on Ayo can be found in [Broline and Loeb 1995]. They generalized Ayo, with each player having n pits, override the Odu rule, and numbered the board in clockwise -n+2, -n+1,….,-1,0,1,….,n,n+1. Their analysis is only confined to the Ayo endgames. They defined a determinable position as an arrangement of stones where it is possible for a player to move such that:

i. A player captures at every turn.
ii. No move is allowed from Odu.
iii. After a player has moved, his opponent has only one stone on his side of the board.
iv. Every stone is captured except the one which is award to his opponent.

Based on these assumptions, Broline and Loeb [Broline and Loeb 1995], also showed a small endgame with nine stones. Thus they provided us with the following lemma.

Lemma (Broline and Loeb)**.** If a player has to move in a determined Ayo position, his stone has to be in pit 1, else in pit 0 if his opponent has to move (pit positions are illustrated in figure 1.).

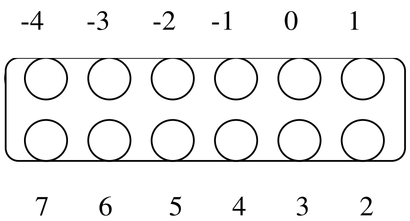

Figure 1: Ayo board labeled with pit numbers

The proof to their lemma is very simple and follows from the definition itself. If the player's opponent has to move, he must capture (i) and leave only one stone (iii), thus simple combinatorics show that it has to be in pit 0 (iv). Thus before the player's move, the stone has to be in pit 1.

Open Problems.

i. Broline and Loeb, have tried to analyze Ayo without Odu, can someone do the same with Odu?
ii. Is it possible, that one can do a more rigorous analysis of Ayo, and give a more generalized Ayo position determination?
iii. Can someone come up with a winning strategy with the maximization of the capturing stones as the goal, from arbitrary positions?

References

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