COMBINATORIAL GAMES: TCHOUKAILLON

Samee Ullah Khan
Department of Computer Science and Engineering
University of Texas at Arlington
Arlington, TX-76019, USA
sakhan@cse.uta.edu

Abstract. Tchoukaillon is a Russian game, played with stones with pits dug into sand. The game is the modern variation of Tchouka which was mostly played in central Europe. Tchoukaillon closely resembles to Ayo (a popular Nigerian game). In this paper we will introduce Tchoukaillon and some basic result about it.

Introduction. Deledicq and Popova [Deledicq and Popova 1977], developed Tchoukaillon as the modern variant of Tchouka, which was played in the ancient times in central Europe. This game also falls in the category of Mancala games [Loeb 1994]. Mancala games are a group of games, with certain common characteristics. They all involve cup-shaded depression called *pits* filled with stones or seeds. Players take turns and maneuver the stones, by various rules, which govern them. Tchoukaillon is a two-player game with defined rules and strategy for players is vital in order to score a win. There is no chance involved; therefore Tchoukaillon falls in the category of *combinatorial games* [Berlekamp et al. 1982].

What is Tchoukaillon? Tchoukaillon is a Russian game, played with stones, with pits dug into sand or soil. The modern version involves playing over a strip of wood also possible is the circular arrangement of the pits. These pits contain a certain number of stones, with one empty pit called *Rouma*, *Cala* or *Roumba*.

The Play. There is no limit as to how many stones are to be used, and neither is a limit on the number of pits [Laguë 1929]. The objective of the game is to put the stones in Roumba. The maneuvering of the stones is called *sow*. Thus like a solitaire game; stones are sown into the empty pit. The sowing takes place as a constant one stone per pit at a time in the direction of Roumba, but it can also be in the opposite direction to Roumba. Therefore there can be only three possibilities during the game:

- If the last stone drops into Roumba, the player has a choice to start sowing another pit of his choice.
- If the last stone drops in an occupied non-Roumba pit, this pit is to be sown immediately.
- If the last pit drops in an empty non-Roumba pit, the game is over and the player who does this losses.

The objective in a two-player Tchoukaillon is to play the last stone in an empty pit so that the next player takes the turn. While doing so he has to sow as many stones in Roumba as possible. The winner of the game is the player with the largest number of stones in Roumba. The Tchoukaillon board is shown in figure. 1.

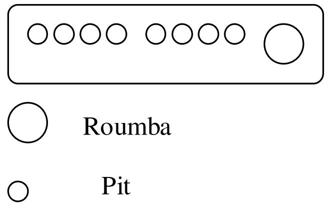

Figure1: Tchoukaillon Board

Known Results. There has not been a lot of work on Tchoukaillon. Loeb first gave some preliminary results on Tchoukaillon [Loeb 1994], later he investigated with Broline [Broline 1995], and related the game with Ayo. They also investigated some winning positions, but disregarded a number of other possibilities of winning and confined themselves to just the objective of sowing stones in Roumba only. Their strategy is based on the results based on the work of [Deledicq and Popova 1977]. They stated the winning move as: "*If a win is possible from a given Tchoukaillon position, the unique winning move must be to harvest the smallest harvestable pit*". The proof to this is very simple. Imagine if there are only two pits on the board, with one pit having less number of stones than the other. If a stone is taken from the pit having more stones, it will increase the number of stones in the smaller pit. Thus the pit at some moment will overflow if a play is continued in this fashion, and an indefinite play will continue. Based on this strategy, they identified the winning positions in Tchoukaillon, by a simple idea of backtracking as many as positions as possible, and came up with the following theorem.

Theorem (Deledicq and Popova)**.** For all s ≥ 0, there is exactly one winning position involving a total of s stones.

Open Problems.

i. Broline and Loeb's theorem has no formal proof in their paper [Broline et al. 1995], rather, they write a program to simulate the results. Can one give a formal proof to their theorem?

ii. Broline and Loeb's strategy confines only to the player's goal to put as many stones in Roumba as possible. What if there are secondary objective too? For instance a trump stone, which if lands as the last stone in Roumba the opponent looses.

iii. Can someone come up with a winning strategy with the maximization of the capturing stones as the goal, from arbitrary positions?

iv. One would also be interested to know, if a break down of Tchoukaillon, would lead to some interesting combinatorial numbers which are often found in Chess, Go and Xiangqi

v. Is it possible, that one can do a more rigorous analysis of Tchoukaillon, and give a more generalized Tchoukaillon position determination?

References

E.R. Berlekamp, J.H. Conway, and R.K. Guy. Winning Ways for your Mathematical Plays I. Academic Press, London, 1982.

D.M. Broline and D.E. Loeb. The combinatorics of mancala-type games: Ayo, tchoukaillon, and 1/π, UMAP, volume 10, number 1, pages 21-36, 1995.

A. Deledicq and A. Popova. "wari et solo: le jeu de calculs africain", Collection Les Distracts, volume 3, Paris, 1977.

D.E. Loeb. Combinatorial properties of mancala, Abstracts of AMS, volume 96, page 471, 1994.

M.A. Sainte- Laguë. Géométrie de situation et jeux. Mémorial des Sciences Mathématiques, Fascicule XLI, Gauthiers-Villars, Paris, 1929.