Approximate Optimal Sensor Placements in Grid Sensor Fields

Samee Ullah Khan
Department of Computer Science and Engineering
University of Texas
Arlington, Texas 76019
Email: sakhan@cse.uta.edu

***Abstract*— This paper proposes a simple heuristic to effectively and efficiently place sensors in grid sensor fields under the constraint of complete coverage. The heuristic guarantees a solution of $\min(1+\alpha, 3)$–optimal when an additional constraint of prioritized placement is enforced, where α is the maximum ratio between the weights (priorities) of the grid points. When there is no prioritized placement, a solution of 2–optimal is guaranteed. We also show that these bounds are the best possible unless P $=$ NP. Comparisons are performed against some well-known sensor placement techniques, where the proposed heuristic outperforms in solution quality and execution time.**

I. Introduction

There are compelling reasons for understanding the interactions between the sensor placement and the data transmission structures, and the effect of these interactions on the efficiency of utilization. Controlled placement will be necessary for applications which have to deploy limited numbers of expensive nodes (e.g. high precision seismic nodes) and hence the location of sensors has to be optimized [6].

One can find numerous sensor placement techniques in the literature. For example, [2] and [3] present a resource–bounded optimization for sensor placement under the constraints of coverage, while [1] employs an additional constraint of minimizing the deployment cost. Recently, [6] proposed a mathematical formulation of the sensor placement problem, with application to target location.

Although, the work discussed above is plausible as it advances the study of sensor placements, yet none of the techniques warrant any theoretical bound on the quality of their placements. Often comparisons are made against random placements (e.g. [2]) which may violate common sense and at times against a brute–force technique (e.g. [3] and [6]) with very small problem instances. Thus, it is imperative to derive an oracle that can provide a certain performance assurance by which one can conclude: "The difference in solution quality between a heuristic and the optimal sensor placement technique."

II. Problem Formulation

An example grid sensor field is shown in Fig. 1.

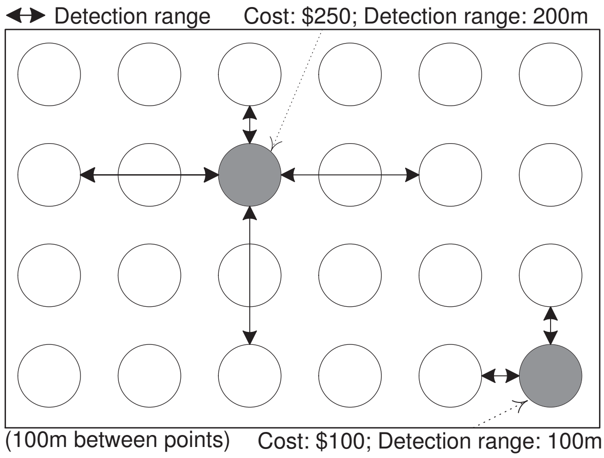

Fig. 1. An example grid sensor field.

The sensor placement (SP) problem can be formulated as a graph theoretical problem. Consider a weighted (explained in the subsequent text) graph $\mathcal{G}=(V,E)$, such that V represents the grid points and E represents the Euclidian distance $c(i,j)$ between each pair of points $i,j \in \{1,2,\cdots,n\}$. Without the loss of generality we assume that $c(i,j) = c(j,i)$. Furthermore, E satisfies triangle inequality ($\triangle$) because of the underlying grid sensor field, i.e., $c(i,j)+c(j,k) \geq c(i,k)$.

Each sensor can sense an intrusion within its detection range r. Many real–life applications have the additional constraint of cost limitations. As noted in [6], for a specialized application (e.g. target location) all sensors have the same deployment cost and thus, this cost constraint indirectly represents the number of allowable sensors, which we represent by k.

For certain applications (e.g. intrusion detection) it may be of interest to give priority to certain grid points. That is, if a grid point $v_i \in V$ resides within a strategic zone, we would like to make sure that a sensor is placed there. (In that case $s_j = v_i$, where s_j represents the jth sensor placed at the ith vertex of $\mathcal{G}$ and $S=\{s_1,s_2,\cdots,s_k\}$.) Such priorities can be represented by associating weights $w(v_i)$ with each of $v_i \in V$.

Formally, the SP problem can be stated as:

Select $S \subseteq V$ as the designated sensors such that each grid point is within r of some $s_i \in S$; the aim of the SP problem is to select a set S of size k.

More specifically,

$$
\begin{array}{lll}
\text{Identify} & S = \{s_1, s_2, \cdots, s_k\} \text{ to} & (1)\\
\text{minimize} & F(x_1, x_2, \cdots, x_k), & \\
\text{where} & F(x_1, x_2, \cdots, x_k) & \\
 & = \max_{v_i \in V} \min_{1 \le j \le k} w(v_i) c(v_i, s_j), & (2)\\
\text{subject to} & \forall\ v_i \in V,\ \exists\ s_j \in S \mid c(v_i, s_j) \le r. & (3)
\end{array}
$$

Briefly, Eq. 1 suggests to selectively place k sensors $S \subseteq V$, such that, preference is given to the grid points that have a higher priority, while minimizing the distance between the sensors for fault tolerance reasons as implied by Eq. 2, subject to the constraint that any grid point $v_i \in V$ is covered by at least one sensor $s_j \in S$ (Eq. 3). The generalized SP problem is known to be NP–complete [1].

III. A Simple Heuristic

Consider the following heuristic for identifying k sensor placement points $v_1, v_2, \ldots, v_k$ of V.

Step 1: Choose $v_1 \in V$ so that $w(v_1) = \max_{v_i \in V} w(v_i)$. (If there is a tie for the maximum, then choose on random.)
Set $D(v_i) \leftarrow w(v_i)c(v_i, v_1)$ for each $v_i \in V$.

Step 2: WHILE $i < k$ DO
Determine v_{i+1} by $D(v_{i+1}) = \max_{v_i \in V} D(v_i)$,
subject to: $c(v_{i+1}, D(v_i)) \le r$.
Set $D(v_i) \leftarrow \min\{D(v_i), w(v_i)c(v_i, v_{i+1}))\}\ \forall\ v_i \in V$.
END WHILE

In essence, the heuristic (acronym H_{sp}) selects any point of the largest weight (priority) for the first sensor's location (Step 1). Then it successfully chooses new locations so that the next location chosen is the grid point which has the largest weighted distance from its nearest sensor within r (Step 2). This process is repeated until all the k sensors are designated.

We take $v_1, v_2, \ldots, v_k \in V$ as the k sensor placements. Notice that $\forall\ v_i \in V$ the *theoretical* solution quality of H_{sp} is measured by $F_{\text{H}_{\text{sp}}} = F(v_1, v_2, \ldots, v_k)$ compared to that of the optimal placement $F_{\text{O}} = F(s_1, s_2, \ldots, s_k)$.

Below, we deduce some important results about H_{sp}.

Theorem 1: H_{sp} takes at most $\mathcal{O}(nk)$ time.

Proof: Step 1 searches a space of n grid points in $\mathcal{O}(n)$. Step 2's complexity is controlled by the while loop which runs at most $\mathcal{O}(k)$ times and within which a similar operation to Step 1 is performed and hence Step 2 takes $\mathcal{O}(nk)$ time. ■

Theorem 2: H_{sp}'s solution at $(k+1)$th iteration is a superset of the solution at kth iteration.

Proof: Let v_{k+1} denote the grid point chosen in a supposed kth iteration of the while loop. (Thus, $v_1, v_2, \ldots, v_{k+1} \in V$ are the grid points that are selected by H_{sp}; k are selected in Step 2 and 1 in Step 1). Let $\tilde{v}_i$ be the grid point such that, when v_i is chosen during the $(i-1)$th iteration of the while loop, $D(v_i) = w(v_i)c(v_i, \tilde{v}_i)$. Then these $D(v_i)$ values form an increasing sequence and $F_{\text{H}_{\text{sp}}} = D(v_{k+1})$. ■

Let $\alpha = \frac{\max_{v_i \in V} w(v_i)}{\min_{v_i \in V} w(v_i)}$ (maximum ratio between the weights of grid points in V) and $\beta = \min(1+\alpha, 3)$, where $3 = \frac{3r}{r} = \frac{c(v_{i+1}, D(v_i))}{r}$ is the worst case upper bound of the optimality on H_{sp}. That is, if a heuristic randomly places sensors at a grid point that is $3 \times r$ of any previously chosen sensor location, it can still satisfy the constraints that are mentioned in (1)–(3). This phenomenon is illustrated in Fig. 2.

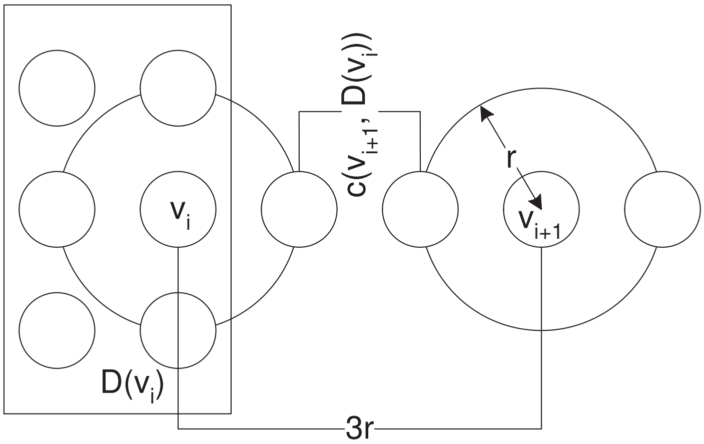

Fig. 2. Worst case upper bound on the optimality on H_{sp}.

Theorem 3: When each grid point is prioritized, H_{sp} grantees a result of β–optimal, i.e., $F_{\text{H}_{\text{sp}}} \le \beta F_{\text{O}}$.

Proof: Let $V_j : \{v_i : c(v_i, s_j) = \min_j c(v_i, s_j)\}$. Clearly, V_j partitions V and $\forall\ v_i \in V_j$, $w(v_i)c(v_i, s_j) \le F_{\text{O}}$. (To prove this we make use of the pigeonhole principle [5, p. 130].) That is, some V_j would include at least two of $v_1, v_2, \ldots, v_{k+1}$. Thus, suppose that V_j contains v_i and v_k, with $1 \le i < k \le k+1$. Let $\gamma = \frac{w(v_k)}{w(v_i)}$ which implies that $\gamma \le \alpha$. (Also observe that $\beta \ge 2(1+\frac{1}{\gamma})$.) Then, we have the following two cases:

(a) $\gamma < 2$ which implies that $\gamma \le \min(2, \alpha)$ and $1 + \gamma \le \beta$.

$$
\begin{array}{lll}
F_{\text{H}_{\text{sp}}} & \le & w(v_k)c(v_k, \tilde{v}_k)\\
 & \le & w(v_k)c(v_k, v_i), \quad \text{since } \tilde{v}_k \text{ is the minimal,}\\
 & \le & w(v_k)\{c(v_k, s_j) + c(v_i, s_j)\}, \quad \text{by } \ddot{\triangle},\\
 & = & w(v_k)c(v_k, s_j) + \gamma w(v_i)c(v_i, s_j)\}\\
 & \le & (1+\gamma)F_{\text{O}}\\
 & \le & \beta F_{\text{O}}.
\end{array}
$$

(b) When $\gamma > 2$, then $\alpha > 2$ which implies that $\beta = 3$. To prove this case, assume that $i > 1$. Let v_l $(l < i)$ be the closest point to v_k when v_i is chosen. Then,

$$
\begin{array}{lll}
F_{\text{H}_{\text{sp}}} & \le & w(v_k)c(v_k, \tilde{v}_k)\\
 & \le & w(v_k)c(v_k, v_l), \quad \text{since } \tilde{v}_k \text{ is the minimal,}\\
 & \le & w(v_i)c(v_i, \tilde{v}_i), \quad \text{else } v_k \text{ would be chosen,}\\
 & \le & w(v_i)c(v_i, v_l), \quad \text{since } \tilde{v}_i \text{ is the minimal,}\\
 & \le & w(v_i)\{c(v_i, v_k) + c(v_k, v_l)\}, \quad \text{by } \ddot{\triangle}. \qquad (4)
\end{array}
$$

At this moment if we assume that $c(v_i, v_k) \le c(v_k, v_l)$, then Eq. 4 would yield $w(v_k)c(v_k, v_l) \le 2w(v_i)c(v_k, v_l)$, which would imply that $w(v_k) \le 2w(v_i)$. But this contradicts $\gamma > 2$. Therefore, the inequality should be $c(v_i, v_k) > c(v_k, v_l)$ and

$$
\begin{array}{lll}
F_{\text{H}_{\text{sp}}} & < & 2w(v_i)c(v_i, v_k)\\
 & \le & 2w(v_k)\{c(v_i, s_j) + c(v_k, s_j)\}, \quad \text{by } \ddot{\triangle},\\
 & = & 2\{w(v_i)c(v_i, s_j) + \frac{1}{\gamma}w(v_k)c(v_k, s_j)\}\\
 & \le & 2(1+\frac{1}{\gamma})F_{\text{O}}\\
 & \le & \beta F_{\text{O}}.
\end{array}
$$

Hence, in both cases we have $F_{\text{H}_{\text{sp}}} \leq \beta F_{\text{O}}$. ∎

Theorem 4: When each grid point has equal priority, H_{sp} grantees a result of 2–optimal, i.e., $F_{\text{H}_{\text{sp}}} = 2F_{\text{O}}$.

Proof: If priorities are same, then $\alpha = \frac{\max_{v_i \in V} w(v_i)}{\min_{v_i \in V} w(v_i)} = 1$ and $\beta = \min(1+\alpha, 3) = \min(1+1, 3) = 2$. ∎

Theorem 5: If $F_{\text{H}_{\text{sp}}} < 2F_{\text{O}}$, then P = NP.

Proof: A close counterpart of the SP problem is the dominating set (DS) problem which is NP–complete. (For a definition of DS see [4, p. 75].) Given an instance of the DS problem, we can define an instance of the SP problem by setting the distance between the adjacent vertices to 1, and non–adjacent vertices to 2. Thus, there should be a DS of size k if and only if $r = 1$ for the SP problem. However, to produce a result of $\beta < 2$ a DS should always identify a solution with $r < 1$, but that is not possible due to the construction of graph $\mathcal{G}$ and the $\ddot{\triangle}$ principle. ∎

IV. Experiments and Discussion

For simulation studies we chose two forms of gird sensor fields: 1) Square (the patterns of which were 100 (10×10), 400 (20×20), 900 $(30\times30), \cdots$, 10000 (100×100) grid points); 2) Rectangular (the patterns of which were 150 (10×15), 300 (15×20), 500 $(20 \times 25), \cdots$, 3300 (55×60) grid points). The Euclidian distance between two adjacent grid points was set to 100m.

As noted in [3] and [6], it is extremely difficult to determine k. To counteract this, we use the concept of power vector (PV) [6]. Each grid point has a power vector PV_i which indicates whether that particular point is covered (1) by any of the sensors or not (0), i.e., $\text{PV}_i = (v_1 \vee v_2 \vee \cdots \vee v_n)$. A sensor field is said to have complete coverage, when every grid point has its power vector equal to one, i.e., $\text{PV} = (\text{PV}_1 \wedge \text{PV}_2 \wedge \cdots \wedge \text{PV}_n) = (1 \wedge 1 \wedge \cdots \wedge 1) = 1$.

Based on the above discussion, we replace the termination condition of the while loop $(i < k)$ with PV $\neq 1$. Accordingly, H_{sp} terminates only when a complete coverage for a grid sensor field is guaranteed.

The *experimental* solution quality of a sensor placement technique under the constraint of full coverage is best measured by observing the sensor density (SD) at which it terminates [6]. We define SD as follows: Let p_i be a boolean variable; 1, if a sensor is placed at a grid point $v_i \in V$ in the sensor field and 0 otherwise. (Note that p_i can easily be measured from the execution of H_{sp}.) Then, $\text{SD}(\%) = \sum_{i=1}^{n} \frac{p_i}{n} \times 100$, where n is the total number of grid points. An effective sensor placement technique would converge to a solution of complete coverage with the minimum possible SD.

For comparisons, we selected four various types of sensor placement techniques. We chose: 1) from [3] the locally greedy technique (PS), 2) from [6] the simulated annealing based algorithm (SA), 3) from [2] the globally greedy approach (MAC), 4) and from [2] the probabilistic technique (MMAC). Notice that none of the chosen comparative techniques cater for the priority based sensor placements. (According to the literature survey this letter introduces the priority based sensor placements for the very first time.) For this purpose, we adjust H_{sp} to determine non–prioritized placements by setting $\forall\ v_i \in V,\ w(v_i) = 1$. Since in all the experiments we kept $r = 100$m, the theoretical lower bound of the optimal SD for complete coverage is 20% — meaning that a sensor can cover at least five grid points (see Fig. 2 for an illustration).

Figs. 3(a)–3(c) summarize the results obtained from a non–prioritized setting. The Outliers and Extremes were limited to 2 and 3 standard deviations, respectively. It can be seen that H_{sp} and SA clearly outperformed (with worst case performance of 26% and 39%, respectively) the rest of the techniques; while PS suffered from its localized view of the problem domain, it terminated 5 times faster than SA.

Next, we observe H_{sp}'s performance on a prioritized sensor field, by defining a priority vector (PrV). For example, PrV = $(20\%, 17\%, 39\%, 24\%)$ suggests that 20% of the (randomly chosen) grid points have no $(w = 1)$, 17% have low $(w = 2)$, 39% have medium $(w = 3)$ and 24% have high $(w = 4)$ priorities, respectively. Fig. 3(d) depicts the recorded results with various randomly chosen PrVs. Due to space limitations, we only depict the results observed over a rectangular sensor field. H_{sp} adapts well to the varying PrVs by proportionally increasing SD with the increase in the number of the grid points. In the worst case, H_{sp} produces a SD $\approx 69\%$ when PrV = $(16\%, 16\%, 16\%, 52\%)$ with $n = 2250$ compared to a SD $\approx 23\%$ when PrV = $(100\%, 0\%, 0\%, 0\%)$ with $n = 2250$.

V. Conclusions

We proposed a simple heuristic to cope with the sensor placement problem with minimal solution quality deviation under the constrains of complete coverage and prioritized placements. Theoretical results were verified by experiments that revealed considerable improvements in the solution quality compared to some well–known techniques.

Possible extensions to this work may include examining the problem of sensor placements with sensors having variable sensing capabilities and under the energy constraints. Such extensions would be very useful for military, where sensors may be required to sense intrusions under hostile environments.

References

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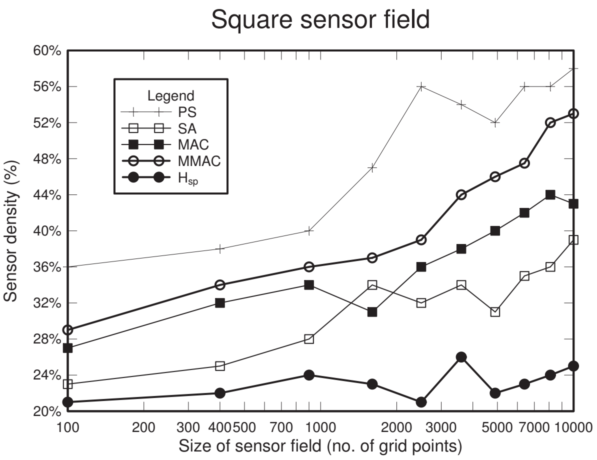
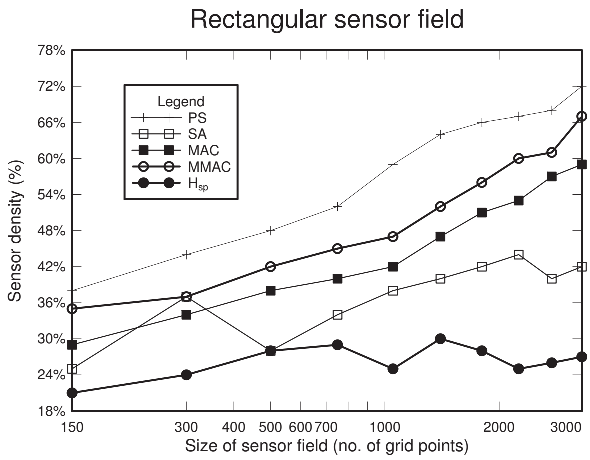
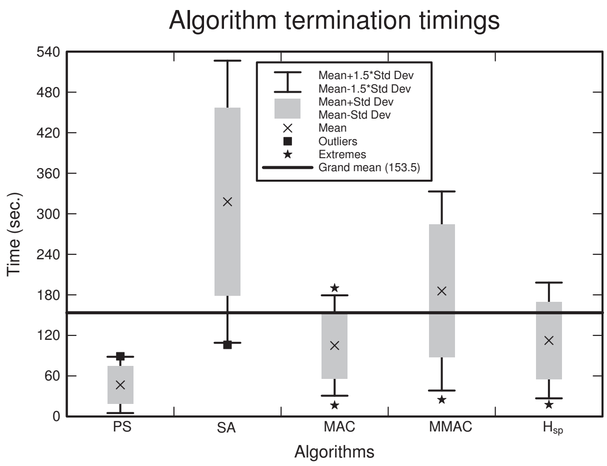


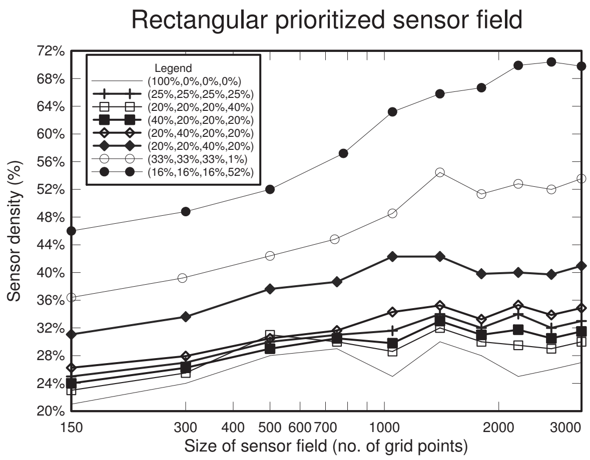
(a) SD vs. size of (square) sensor field.



(b) SD vs. size of (rectangular) sensor field.



(c) Algorithm termination timings.



(d) SD vs. size of (prioritized) sensor field.

Fig. 3. Simulation results.