Heuristics-based Nominal Channels Allocation in Cellular Networks

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ABSTRACT

In this paper, we develop four heuristics to deal with the nominal channel allocation in cellular networks. We provide the theoretical bound on the number of channels for voice and data services. The four heuristics are compared using varying traffic distributions.

Keywords

Channel allocation problem, channel capacity, heuristics, optimization

1. INTRODUCTION

Since the development of cellular networks, the demand for multiple services has continuously increased. To meet the future demand, many new modulations and techniques have been proposed. However, in a given system, the traffic capacity of the whole cellular network will always be influenced by how the channels are allocated. A proper allocation can significantly reduce the number of channels to meet the traffic demand.

In this paper, we investigate the channel assignment scheme for cellular networks as a dual optimization problem. This is significantly different from previous approaches, such as [1] and [2] that have focused on either voice services or data services. We suspect that this dual optimization problem is in the class NP. Therefore, efficient and effective heuristics are required to compute near optimal solutions.

2. PROBLEM FORMULATION

The problem formulation consists of two parts: (a) voice and (b) data services.

2.1 Channel allocation for voice service

Suppose the total number of available channels in a cellular network of N cells is M. We use λ_i to denote the voice traffic demand in cell C_i. Let m_i be the number of channels for voice service. The call blocking probability in the cell is given as:

$$B(\lambda_i, m_i) = \sum_{k=0}^{m_i} [\frac{\lambda_i^k}{k!}]^{-1} \frac{\lambda_i^{km_i}}{(m_i!)}. \quad (1)$$

To ensure the quality of the calling services in the network, the blocking probability in each cell must be no greater than R_0. This also ensures that the average blocking probability in the system must be no greater than R_0. However, if one channel is used simultaneously by the neighboring cells, then severe inter-cell interference may occur. Based on Refs. [2] and [3], we can assume that the interference only happens when the same channel is reused by the adjacent cells. In this case, if the radius of the cell is *r*, then the minimum reuse distance for a channel is *2r*.

To minimize the total number of channels, we introduce the concept of the patterns. A pattern is a set of cells in which each cell simultaneously uses the same channels without causing any interference. Each cell belongs to one and only one pattern. Assume that we have P patterns and x_i denotes the number of channels allocated to pattern P_i. Therefore, the channel assignment problem that minimizes the total number of channels for voice service

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subject to the average blocking probability can be given:

$$min \sum_{i=1}^{P} x_i, \tag{2}$$

subject to

$$B(\lambda_i, m_i) \leq R_0; \tag{3}$$

$$m_i < M \; \forall i, (1 \leq i \leq N). \tag{4}$$

2.2 Channel allocation for data services

After the first step, the total remaining number of channels available for data services has been fixed as m_0 $(m_0 \neq M)$. We need to judicially allocate these channels so that the total traffic capacity of data service throughput is maximized.

The voice service has a very low requirement for the communication rate [4]. However, the channels used for data services must have high communication rate to fulfill the QoS constraints. Therefore, we must choose the best m_0 of M channels for transmitting data. Let $M_0, M_1, \cdots, M_N$ denote the number of the users in every cell and the total number of users be denoted by $M_C = \sum_{i=1}^{N} M_i$. We assume that the signal to noise ratio (SNR) is known a *priori*.

$$SNR_{ki} = \frac{P|h_{ki}|^2}{N_0}, \tag{5}$$

where SNR_{ki} is the SNR of user k using channel H_i, h_{ki} is the path loss, and N_0 is the noise. This assumption is reasonable because in a cellular system, the channel state information (CSI) is available through the uplink feedback. Therefore, for a given network, the goal must be to maximize the total capacity C for data services as follows:

$$C = \sum_{k=1}^{M_C} \sum_{i=1}^{m_0} log_2(1 + SNR_{ki}) \times y_{ki}, \tag{6}$$

subject to the following two constraints:
(a) $y_{ki} = 1$ only if channel H_i is allocated to user k, otherwise 0;
(b) $\forall i \in \{1, 2, \cdots, m_0\}$, if $y_{ki} = 1$, then $y_{k'i} = 0$ for any two users k, k' in the same cell.

From Eq. (4), the total capacity is only related to y_{ki}. This is due to the fact that SNR_{ki} for each user is known.

Because the traffic demand in a cell is directly proportional to the user population. Therefore, the traffic demand in a cell may be represented as a function of the cell's user population [5]:

$$ln\rho(r) = ln\rho_0 + a\sqrt{r}, 0 \leq r \leq L, a < 0, \tag{7}$$

where $\rho(r)$ is the population density at distance r from the center, ρ_0 is the population density at the center of each cell, a is the rate at which the logarithm of density decreases with the square root of the distance to the center, and L is the maximum distance from the center to the edge.

3. THEORETICAL ANALYSIS

In the subsequent text, we derived the upper and lower bounds on the the number of channels used for voice and data services. Because, theoretically, voice and data services can be used interchangeably, deriving a bound for voice service will be sufficient. First, the upper bound is derived based on map coloring theorem.

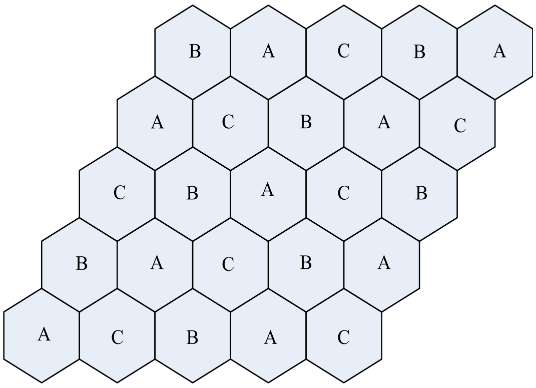

Figure 1: An example of 5×5 three coloring cellular network.

We first select three neighboring cells and assign them into three patterns A , B and C. Then we search the nearest cells that can be added to each pattern without causing interference. This procedure is repeatedly applied for each pattern until all cells in the network are in patterns. For each pattern, we allocate the minimum number of channels to fulfill the QoS constraints. Then the total number of channels can be given as:

$$\sum_{i=A,B,C} argmin\{m_i | B(\lambda_i, m_i) \leq R_0\}. \tag{8}$$

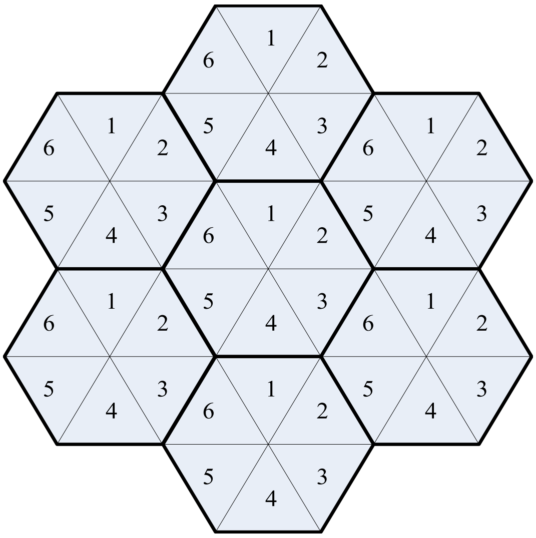

Figure 2: Division of cells into 6 regions for deriving the lower bound

Next, we derive the lower bound. We divide each cell into 6 regions as in Fig. 2. The distance between regions with same regional number is the reuse distance. Suppose users are equally distributed in every cell, and the maximum traffic demand of all cells is λ_0. The total number of channels becomes:

$$6 \times argmin\{m | B(\lambda_0, m) \leq R_0\}. \tag{9}$$

4. PROPOSED APPROACH

4.1 Greedy Algorithms

We proposed the following two greedy heuristics: (a) GreedyMAX in which the cell with the maximum traffic demand is given preference so that channels can be reused frequently and (b) GreedyMIN in which the cell with the minimum traffic demand is given preference so that channels can be reused flexibly. The GreedyMAX pseudo-code is given in Algorithm 1. Note that $D(g,k) = \sqrt{(k_1-g_1)^2+(k_1-g_1)(k_2-g_2)+(k_2-g_2)^2}$ is the distance between cells C_g and C_k measured from their centers.

```
Input: $I$, $\{\lambda_1, \lambda_2, \dots, \lambda_N\}$, $r$, $R_0$
Output: min number of channels $x$
Initialization $I = \{1, 2, \dots, N\}$,$x = 0$,$i = 0$;
while $I \neq \emptyset$ do
    $i = i + 1$;
    $I_P(i) = \{k|min(\underset{k}{\text{argmax}}\,\lambda_k), k \in I\}$;
    $I = I - I_P(i)$;
    $x_i = \underset{x}{\text{argmax}}\{B(\lambda_k, x) \leq R_0, k \in I_P(i)\}$;
    $I_R = \{g|D(g,k) \geq 2r, k \in I_P(i), g \in I - I_p(i)\}$;
    while $I_R \neq \phi$ do
        $g' = \underset{g \in I_R}{\text{argmax}}\,\lambda_g$;
        $I_P(i) = I_P(i) \bigcup \{g'\}$;
        $I_R = \{g|D(g,k) \geq 2r, k \in I_P(i), g \in I - I_p(i)\}$;
    end
    $x = x + x_i$;
    $I = I - I_P(i)$;
end
return x
```

Algorithm 1: GreedyMax Algorithm

For GreedyMIN, we only need to adopt following changes in Algorithm 1: (a) Line 4 of Algorithm 1 must be replaced with $I_P(i) = \{k|min(\underset{k}{\text{argmin}}\,\lambda_k), k \in I\}$, and (b) Line 9 must be replaced with $g' = \underset{g \in I_R}{\text{argmin}}\,\lambda_g$.

4.2 Genetic Algorithm

The genetic algorithm has the following 4 steps: (a) selection, (b) crossover, (c) mutation, and (d) evaluation.

First, we randomly select two solutions $S_A = \{\overrightarrow{e}_A(1), \overrightarrow{e}_A(2), \cdots, \overrightarrow{e}_A(N)\}$ and $S_B = \{\overrightarrow{e}_B(1), \overrightarrow{e}_B(2), \cdots, \overrightarrow{e}_B(N)\}$ from the initial population that is limited to 100 solutions, where $\overrightarrow{e}(i) = \{a_0, a_1, \cdots, a_M\}$. a_M equals to 1 only if the channel M is allocated to cell i, otherwise it will be zero. Then we do the crossover and the mutation for S_A and S_B to get the new solutions:

$S'_A = \{\overrightarrow{e}_A(1), \overrightarrow{e}_A(2), \cdots, \overrightarrow{e}_A(\lfloor N/2 \rfloor), \overrightarrow{e}_B(\lfloor N/2 \rfloor + 1), \cdots, \overrightarrow{e}_B(N)\}$,

$S'_B = \{\overrightarrow{e}_B(1), \overrightarrow{e}_B(2), \cdots, \overrightarrow{e}_B(\lfloor N/2 \rfloor), \overrightarrow{e}_A(\lfloor N/2 \rfloor + 1), \cdots, \overrightarrow{e}_A(N)\}$.

Next, we evaluate the new solutions to check if they meet the constraints. If the new solutions meet the constraints, then we add them to the solution set. Otherwise, we use the fix function to new solutions and evaluate them again. If the fixed solutions can meet the constraints, then we add them to the solution set. If not, we discard them and generated two new solutions again.

Generally, the failure to meet the constraints for new solution mainly happened in the $\overrightarrow{e}_A(\lfloor N/2 \rfloor)$ and $\overrightarrow{e}_B(\lfloor N/2 \rfloor + 1)$. The fixfunction can be implemented as follow:

```
Search (Neighborhood($\overrightarrow{e}_A(\lfloor N/2 \rfloor)$));
if channels available$\neq 0$ then
    add available channels to $\overrightarrow{e}_A(\lfloor N/2 \rfloor)$;
end
update $\overrightarrow{e}_A(\lfloor N/2 \rfloor)$
```

Table 1: Performance of different heuristics in uniform distributed network

heuristics	m_0	m_s	C	O
GreedyMAX	64	431	49.7	4.231
GreedyMIN	66	428	47.2	4.781
Genetic Algorithm	58	459	61.4	19.342
Dynamic Programming	54	468	70.3	23.292

4.3 Dynamic Programming

Because the lower bound of the number of channels is obtained based on pattern, we may use dynamic programming to get the optimal solution since it is also based on patterns. We denote $f(n)$ as the minimum number of the channels used for first n cells. $t(n)$ is the number of available channels in cell C_n that have been used by cells $C_1, C_2, \cdots, C_n$. Suppose the number of channels needed in cell n is m_n, and $T(n)$ is the number of new channels added in cell n. $T(n)$=$m_n - t(n-1)$ if $m_n \geq t(n-1)$, otherwise $T(n) = 0$. Therefore, the function $f(n)$ can be expressed as: $f(n) = f(n-1) + T(n)$.

5. SIMULATION RESULTS

In the simulation, we considered a cellular network, which consists of $N = 49$ cells. Assume the total number of the channels used for calling no exceed than 100 and there are 150 channels available in the cellular network. The blocking probability threshold in each cell is equal to $R_0 = 0.05$. The arrival of the calls is a Poisson process and the duration of the call is exponentially distributed with the mean of 3 minutes. We suppose the SNR for users in the cellular network is $10db$.

In a network with uniform traffic distribution of 80 calls/h in all 49 cells, the results are presented in Table I. m_0 is the total number of channels used for voice service. m_s is the simultaneously usable channels for voice service. C is the maximum capacity versus reached by using the remaining channels and O is the running time of the algorithm.

In Table II, the results are shown for a non-uniform distributed network in which the traffic demand of cells is randomly distributed between 20 to 120.

From the two tables, we observe that dynamic programming and genetic algorithm have better performance in both cases. In the uniform distribution case, the theoretical lower bound for the number of the channels used in voice data is 52. The result of dynamic programming is quite close to it.

Table 2: Performance of different heuristics in nonuniform distributed network

heuristics	m_0	m_s	C	O
GreedyMAX	80	490	32.7	6.045
GreedyMIN	82	484	30.8	6.396
Genetic Algorithm	72	514	53.1	28.571
Dynamic Programming	66	528	64.1	37.729

6. CONCLUSION

In this paper, we solved the channel allocation problem in cellular network as a dual optimization problem and four heuristics are proposed. The simulation results show that dynamic programming and genetic algorithm have better performances and dynamic programming can approach the lower bound.

7. REFERENCES

[1] S. Kim, K. Chang, S. Kim, “A Channel Allocation for Cellular Mobile Radio Systems Using Simulated Annealing," *Telecommunication Systems.*, vol. 14, no. 1, pp. 95-106, Aug. 2000.

[2] G. Li and H. Liu, “Downlink Radio Resource Allocation for Multi-Cell OFDMA System," *IEEE Transactions on Wireless Communication*, vol. 5, no. 12, pp. 3451-3459, Dec. 2006.

[3] R. Y. Chang, Z. Tao, J. Zhang, and C.–C. J. Kuo, “Multi-Cell OFDMA Downlink Resource Allocation Using A Graphic Framework," *IEEE Transaction on Vehicular Technology*, vol. 58, no. 7, pp. 3494-3507, Sep. 2009.

[4] A. Goldsmith, *Wireless Communications.* New York: Cambridge University Press, 2005.

[5] J. B. Parr, “A population–density approach to regional spatial structure," *Urban Studies*, vol. 22, pp. 289-303, 1985.