Smart Data Center

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1 Introduction

All the internet services available these days are dependant and running in data centers. Companies like Google, Facebook, and Microsoft hosts millions of servers in their data centers to provide services to their users [19]. The enormous size of data centers leads to huge energy consumption. According to a news article, Google drew 260 MW of power in 2011 [6] that cost millions of dollars.

Recently, the researchers have focused on reducing the data center energy cost. The researchers have focused on migration of the workload from one geographical location to another to use the time and location dependent electricity prices [2] [21]. Similarly, researchers have also focused on the use of Uninterrupted Power Supply (UPS) in data centers to shave off the peak power demands [24]. UPS has also been used to safe the data center from the unexpected power outages. The power outages also cost millions of dollars to data centers. Amazon was hit a severe power outage in 2012 that cost Amazon millions of dollars [17].

The modern smart grid provides the needed electricity to the data centers. Smart grids provide different pricing schemes for electricity based on different time scales [10, 18]. Due to huge electricity demands, the data centers acquires electricity from grids using long term contracts in day ahead market. The long term contracts cost lower than the real time market price of electricity [18]. In this paper, we propose the idea to buy electricity from more than one smart grid. The local power grid will act as the main power source for the data center. However, data center will also be powered by the remote grid with the surplus power. The data center can purchase the available surplus power from remote grids at lower prices than local grid long term

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market and real time market prices. The sale of surplus energy is advantageous to the remote grids as the surplus energy is mostly wasted [14]. The amount of available surplus power could vary over time. UPS available in the data center for backup can be used to store surplus power from the remote grid or when price from the local grid is low. When the surplus power from the remote grid is insufficient or the price of electricity at local grid is high, the stored power in the UPS batteries can be used.

In this work, we have targeted the key problem in the data center that how to minimize the long term running cost of the data center? Several sub problems are investigated to answer the key problem. How much power should be purchased from the local grid in long term and real time price rates? How to efficiently use the available surplus power from the remote grid? How to best use the UPS for power saving and for backup while saving the life of battery for longer time? To optimally utilize the data center with multiple sources while minimizing the operational cost is really challenging task. There are numerous uncertainties both in power demand and supply side. The power demands of the data center are time varying and job dependent. Each job can consume different amounts of power as they may utilize different number of machines. Similarly on the supply side, availability of surplus energy is an uncertain and long term and real time prices from local grids can change with time.

Previous works on reducing the power consumption and cost of electricity for data center, assume the prior knowledge of the power demand to predict the future power demands [25]. The previous works do not consider the scenario of providing the power to the data center from multiple power grids. In contrast to the previous works, we aim to design efficient strategy to reduce the long term operational cost of the data center while having the constraints of dynamic power demand with no previous knowledge and uncertain availability of surplus power from remote grid.

We develop an algorithm titled "Smart Data center" to make a data center smarter using two stage Lyapunov optimization techniques. Smart Data center computes the amount of power to be purchased from the local grid in a long term contract. The amount of electricity to be purchased from the local grid on real time market rate and amount of the electricity to be stored and retrieved from the UPS are also computed by the Smart Data center algorithm. We analyze the performance of the Smart Data center algorithm through rigorous theoretical analysis in this work.

2 System Model

We assume a discrete time model for the working of a data center. The notations and their meanings in the model are presented in the Table 1. Time for the model is divided into k slots each of length T. The length T depends on the intervals provided by the grids in long term contracts. Each time slot is further divided into fine grained slots of length L. We also assumed that power demand of the data center $d(t)$ and available surplus power of remote grid $r(t)$ are random variables. The operations of the data center in a system model include following key decisions.

Table 1 Notations and their meanings

Notation	Meaning
t	Coarse grain time slot
τ	Fine grain time slot
$d(t)$	Power demand
$r(t)$	Surplus power at remote grid
$Pmax$	Maximum purchasing power capacity
$E_{full}(t)$	Electric power units purchased in long term for time t
$E_{lt}(t)$	Electric power units purchased in long term for time τ
$r(t)$	Units of surplus power purchased from remote grid
$p_{lt}(t)$	Unit price of electricity in long term market
$p_{rt}(t)$	Unit price of electricity in real time market
$p_s(t)$	Unit price of surplus power from remote grid
P_{grid}	Maximum capacity of local grid
$E_u(\tau)$	Level of power in the UPS
$E_u\ max$, $E_u\ min$	Maximum and minimum capacity of UPS
$D(\tau)$	Amount of power discharge from battery at time τ
$R(\tau)$	Amount of power (Re)charged in battery at time τ
η	Efficiency of the UPS

2.1 Long Term Power Purchase

The data center takes notes of the power demand *d(t)* and available surplus power at the remote grid *r(t)* at the start of each coarse grained time slot *t*. The data center is provided with a maximum threshold limit *Pmax* as a maximum purchasing power capacity. Based on the observations, the data center takes the decision that how much electric power units E_{full} *(t)* should be purchased from the local grid at price $p_{lt}(t)$ within the purchasing capacity at the start of coarse grain time slot. After the purchase, the data center divides the electric power units equally to be used in all the fine grain time slots.

$$E_{lt}(t) = \frac{E_{full}(t)}{L}. \tag{1}$$

For example, suppose the data center decides to purchase 720 *KW* when the length of the coarse grained time slot is one day and fine grained time slot is 1 h. In the above mentioned case, the data center will distribute the 500 *KW* equally, i.e., 500/24 = 30 *KW* for each fine grained time slot.

2.2 Real Time Power Purchase

We have assumed that the cost of the surplus power at the remote grid is lower than the local grid long term and real time power purchase. Whenever there is a surplus power available on the remote grid in time slot t, the data center tries to use it as much as possible. In case when surplus power is more than the power demand, the excess power is used to charge the UPS. At each fine grained time slot τ, the UPS will not be needed to charge or discharge if the sum of long term power purchase from local grid and surplus power from the remote grid is less than the total power demand from the data center.

$$E_{lt}(t) + r(\tau) \geq d(\tau). \tag{2}$$

Otherwise, if the power demand is more than the sum (left hand side of the Equation) than the data center has to make the decision to discharge the power from the batteries $D(\tau)$ of the UPS. If the UPS power is not enough for the remaining power demand, more electric power units $E_{rt}(\tau)$ are purchased from the local grid at real time price rate $p_{rt}(\tau)$ To balance out the equation, any surplus purchased power is used to charge the batteries of the UPS C(τ). We have an overall equation of the data center as

$$\begin{gathered} E_{lt}\ (t) + \ E_{rt}(\tau) + D(\tau) + r(\tau) - C(\tau) = d(\tau), \\ 0 \leq E_{lt}\ (t) + E_{rt}(\tau) \leq Pgrid. \end{gathered} \tag{3}$$

3 Constraints

There are a number of constraints that must be satisfied by the data center.

3.1 Purchasing Accuracy and Cost

The price of surplus electricity from the remote grid is lower than the electricity prices in the long term contract and real time market price rates from local grids.

$$p_{rt}(\tau)\ >\ p_{lt}\ (t) > p_s(\tau). \tag{4}$$

However, availability of surplus electricity from the remote grid is dynamic in nature. Similarly, the data center can purchase electricity from real time market but that is the most expensive. Therefore, the data center has to make a decision of purchase of electricity with accuracy to keep the overall cost of the electricity purchased to be minimized.

3.2 Data Center Availability

Let $E_u(\tau)$ be the level of the power in the UPS batteries at time τ. Power in the batteries of the UPS is affected by the efficiency of USP (dis)charging. We assumed that efficiency for discharging and charging $\eta \in [0,1]$ is same. The dynamics of the UPS power level can be expressed by the following equation

$$E_u(\tau + 1) = E_u(\tau) + \eta R(\tau) - \frac{D(\tau)}{\eta}. \tag{5}$$

To guarantee the availability of the data center in case of power outages, minimum level of power must be maintained in the batteries of the UPS. If the maximum power storage capacity of the UPS is $E_u\ max$ than we have

$$E_u\ min\ <\ E_u(\tau) < E_u\ max. \tag{6}$$

3.3 UPS Lifetime

At given time t, the amount of power that can be stored or retrieved from the batteries of the UPS is limited by their maximum amounts

$$0\ \leq D(t) \leq D\ max, 0\ \leq R(t) \leq R\ max. \tag{7}$$

The lifetime of the UPS is constrained within the number of cycles of UPS charging and discharging [24]. The operating cost of the UPS also depends upon UPS charging and discharging cycles. We assume that cost of UPS C_r is same in both cases of charging and discharging. If the purchase cost of a new UPS is $C_{purchase}$ that can sustain M_{cycles} than we have

$$C_r = \frac{C_{purchase}}{M_{cycles}}. \tag{8}$$

If the life of UPS is defined as Life, than the maximum number of times the batteries of the UPS are allowed to charge and discharge over a longer period of time [0, t − 1] and $t \in kT$, will be

$$N_{max} = \frac{M_{cycles} * kT}{Life}. \tag{9}$$

The variable kT is the total time for modeling, i.e., k coarse grain slots of length T. Therefore, N_{max} satisfies the following equation

$$0 \leq \sum_{\tau=0}^{t-1} \partial(\tau) \leq N_{max}. \tag{10}$$

In the above equation, $\partial(\tau)$ denotes the usability of the batteries of UPS in time τ. The variable $\partial(\tau)$ will be 1 if the discharge or recharge occurs otherwise the variable takes the zero value. The operational cost of the UPS can now be calculated as the product of usability of the batteries of UPS and cost of UPS in time slot t.

$$Cost\ of\ UPS_{opertional} = \partial(t) * Cr. \tag{11}$$

4 Cost Minimization

The operational cost of the data center at a fine grained time slot τ is the sum of the costs for purchasing electricity from the local grid, remote grid, and the operational cost of the UPS.

$$\begin{aligned} &Cost\ of\ data\ center_{opertiaonal}(\tau) \\ &= E_{lt}\ (t) p_{lt}\ (t) + \ E_{rt}(\tau) p_{rt}(\tau) + r(t) p_s(t) + \partial(t) Cr. \end{aligned} \tag{12}$$

In this work, we aimed at designing the algorithm that can make decisions by solving the following minimization problem

$$\min Cost\ of\ data\ center_{avg} \cong \lim_{t\to\infty} \begin{array}{c} \frac{1}{t}\sum_{\tau=0}^{t-1} Cost\ of\ data\ center_{opertiaonal}\ (\tau), \\ \forall t\text{: } Constraints\ \ (3)\ (6)\ (7)\ (8) \end{array} \tag{13}$$

5 Algorithm Design

We design our algorithm using the Lyapunov optimization technique to achieve the near optimal solution. The algorithm does not use the prior knowledge of power demand. To guarantee the availability of data center, the algorithm has to track the status of power level in the batteries of the UPS. Tracking the status of power level in the batteries is necessary as we want to ensure that each time the power is discharged or charged from the batteries of the UPS, there should be enough power remain in the battery that can be used during blackouts as backup. To track the battery power of the UPS, we use the supporting variable *X(t)* defined as follows:

$$X(t) = E_u(t) - \frac{V P_{max}}{T} - \ E_u \min - \frac{D\, max}{\eta}. \tag{14}$$

In the above equation, V is a control variable that ensures that whenever batteries of UPS is charged or discharged, the power in the batteries should lie in the minimum and maximum level. With increment in the time slot t, the variable *X(t)* changes as

$$X(t+1) = X(t) + \eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}. \tag{15}$$

We consider the constraint of availability of power level in the batteries of the UPS as a queue problem and transform the constraint into queue stability problem, similar to the work presented in [23]. We define the Lyapunov function to represent the scalar metric of queue congestion as

$$L(t) \cong \frac{1}{2} X^2(t). \tag{16}$$

We use the Lyapunov drift to stabilize the system that pushes the Lyapunov function towards lower congestion state. The Lyapunov drift over time period T is defined as

$$\Delta LD_T \ \cong L(t+T) - L(t)|X(t). \tag{17}$$

We obtained the drift penalty term by following the Lyapunov drift penalty framework [5]. In every time frame of length T, the Smart Data center algorithm makes a decision to minimize the upper bound on the drift plus penalty. The upper bound can be obtained by adding the operational cost to the drift plus penalty as:

$$\Delta LD_T(t) + V * \sum_{\tau=0}^{t+T-1} \mathit{Cost\ of\ data\ center}_{opertiaonal}\ (\tau) \mid X(t). \tag{18}$$

The data center chooses the control parameter V to adjust the tradeoff between the level of power in the UPS for backup and minimizing the operational cost of the data center. For optimal cost minimization, V has to be set high and for more power back up, the value of V needs to be small.

5.1 Drift Plus Penalty Upper Bound

A key question is to find out the upper bound for the value of V. The upper bound of drift plus penalty helps in finding the maximum operational cost of the data center that can be saved under the constraint of keeping the power in the batteries of the UPS for backup. To find out the upper bound we assume that Lyapunov function $L(0) > \infty$, $t = KT$, $\tau \in [t, t+T-1]$, and $V > 0$. We take the squares of the Eq. (15) on both sides.

$$X(\tau+1) = X(\tau) + \eta R(\mathrm{t}) - \frac{D(\tau)}{\eta},$$

$$X^2\ (\tau+1) = X^2(\tau) + 2 * X(\tau) * \left[\eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}\right] + \left[\eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}\right]^2,$$

$$\frac{\left[X^2\ (\tau+1) - X^2(\tau)\right]}{2} = X(\tau) * \left[\eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}\right] + \frac{\left[\eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}\right]^2}{2}.$$

As $R(t) \in [0, R_{max}]$ and $D(t) \in [0, D_{max}]$, the above equation is transformed into the following equation

$$\frac{\left[X^2\ (\tau+1) - X^2(\tau)\right]}{2} \le X(\tau) * \left[\eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}\right] + \frac{1}{2} max \left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]. \quad (19)$$

We get the 1-time slot conditional Laypunov drift by taking the expectation over power demand, available surplus power and its price in the remote grid, and the price of the electricity in long term contract and real time market in the local grid on the auxiliary variable *X(t)* as

$$\Delta LD_1(t)\ \le X(\tau) * \left[\eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}\right] + \frac{1}{2} max \left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]. \quad (20)$$

By taking the sum of all inequalities over $\tau \in [t, t+1, \ldots\ldots\ t+T-1]$, we obtain the T-time slot Laypunov drift

$$\Delta LD_T(t)\ \le X(\tau) * \left[\eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}\right] + T * \left\{\frac{1}{2} max \left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta 2}\right]\right\}. \quad (21)$$

Finally we add the operational cost on both sides of the equation and get the upper bound on the T-time slot Lypunov drift plus penalty.

$$\begin{aligned} \Delta LD_T(t) + V * \sum_{\tau=0}^{t+T-1} Cost\ of\ data\ center_{opertiaonal}(\tau)\ \mid X(t) \\ \le X(\tau) * \left[\eta R(\mathrm{t}) - \frac{D(\tau)}{\eta}\right] + T \\ * \left\{\frac{1}{2} max \left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]\right\} + V \\ * \sum_{\tau=0}^{t+T-1} Cost\ of\ data\ center_{opertiaonal}(\tau)\ \mid X(t). \end{aligned} \quad (22)$$

The Smart data center algorithm follows the drift plus penalty principle and tries to minimize the right hand side of the Equation.

5.2 Relaxed Optimization

In order to minimize the right hand side of the Eq. (22), the data center needs to know the queue backlog *X(t)* over time $\tau \in [t, t+T-1]$. The amount of available surplus power in the remote grid, the power level in the batteries of the UPS, and the power demand affects the queue *X(t).* Moreover, the dynamic nature of electricity prices, available surplus power, and power demand are major constraints for taking the decision. The researchers have used forecasting techniques to predict the variable nature of the parameters. However, one day head forecasting techniques causes daily mean errors of approximately 8.7 % [16]. Therefore, in order to remove the need of forecasting techniques we used the near-future queue blog statistics. We used the current values of the queue, i.e., $X(\tau) = X(t)$ for the time period $t < \tau \leq t+T-1$ for the backlog statistics. However, the use of near future queue backlog result in slightly "loosening" of the upper bound on the drift plus penalty term. We have proved this loosening of the upper bound in Corollary 1.

Corollary 1 (Loosening Drift plus penalty bound) Suppose the control parameter V is positive and for some nonnegative integer K, the time slot t is equal to KT. By changing the time period from τ to t in the queue X, the drift plus penalty satisfies:

$$\Delta LD_T\,(t) + V\mathbb{E}\{\sum_{\tau=t}^{t+T-1} \textit{Cost of data center}_{opertiaonal}\,(\tau)\,|X\,(t)\}$$

$$\leq \left\{ \frac{1}{2} max \left[\eta^2 R^2\,(\mathrm{t}),\ \frac{D^2\,(\tau)}{\eta^2}\ \right] \right\}$$

$$+ \frac{T\,(T-1)\left[\eta^2 R^2\,(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]}{2}$$

$$+\ \mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} X\,(t) * \left[\eta R(\tau) -\ \frac{D\,(\tau)}{\eta}\right] |X(t) \right\} \tag{23}$$

Proof According to the Eq. (15), for any $\tau \in [t, t+T-1]$, we get

$$X(t) - \frac{(\tau - t)Dmax}{\eta}\ \leq X(\tau),$$

$$\text{and } X(\tau) \leq (\tau - t)\eta Rmax.$$

Therefore, recalling each term in Eq. (22), we have

$$\sum_{\tau=t}^{t+T-1} X(\tau)\,[R\,(\tau)\ \ \eta - \mathrm{D}\,(\tau)/\eta]$$

$$\leq \sum_{\tau=t}^{t+T-1} [X(t) + (\tau - t)\,\eta Rmax]\,R(\tau)\eta$$

$$
\begin{aligned}
&- \sum_{\tau=t}^{t+T-1} \left[X(t) - (\tau - t)\, Dmax/\eta\right] D(\tau)/\eta \\
&=> \sum_{\tau=t}^{t+T-1} X(\tau)\left[R(\tau)\,\eta \ - D(\tau)\,/\eta\right] \\
&+ \sum_{\tau=t}^{t+T-1} \left[(\tau - t)\eta^2 Rmax\, R(\tau) - Dmax\, D(\tau)/\eta^2\right] \\
&\leq \sum_{\tau=t}^{t+T-1} X(\tau)[R(\tau)\ \ \eta - D(\tau)/\eta] \\
&+ \frac{T(T-1)\left[\eta^2 R^2\,(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\ \right]}{2}
\end{aligned}
$$

By substituting the above inequalities into Eq. (22), the corollary is proved.

5.3 Two Timescale Smart Data Center Algorithm

We see that the upper bound that can be achieved using Eq. (23) is larger than the one in Eq. (22). The Smart Data center algorithm aims to make the decision to minimize the right hand side of the Eq. (23). Depending on the available surplus power at the remote grid *r(t)*, the algorithm has to make the decision to purchase $E_{full}(t)$ at the start of the each coarse grained timeslot t. Moreover, at the beginning of each fine grain time slot τ, the Smart Data center algorithm has to make the decision for $E_{rt}(\tau), D(\tau), and R(\tau)$. Consequently, the problem can be separated into two timescales as two subproblems. In the coarse grain time slot, the algorithm has to make the decision to ensure that current energy demand is fullfiled and batteries of the UPS should be charged with enough power for the future use. The decisions for UPS charging and discharging along with purchase of electricity on real time rate from the local grid are made by algorithm at the start of each fine grain timeslot. The queue statistics are updated at the end of each time slot.

Algorithm 1 The Smart Data center Algorithm

1. *Long term planning*: The data center decides the optimal power purchase $E_{full}(t)$ at the start of each coarse-grained time slot $t = kT$ where k is nonnegative integer. The long term ahead power purchase is to minimize the following problem

$$
\begin{aligned}
\min \mathbb{E} &\left\{ \sum_{\tau=t}^{t+T-1} V\left[E_{lt}(t)p_{lt}\,(t) + E_{rt}(\tau)p_{rt}\,(\tau) + r(\tau)p_s\,(\tau)\right] |X(t) \right\} \\
&+ \mathbb{E}\left\{ \sum_{\tau=t}^{t+T-1} X(\tau)[R(\tau)\,\eta - D(\tau)/\eta]|X\,(\mathrm{t}) \right\} \\
&s.t.\ \ (3)
\end{aligned}
$$

2. *Real time power balancing*: The data center divides the power purchased in long term equally $E_{lt}(t) = \frac{E_{full}(t)}{L}$ among all the fine grained time slots τ ε [t, t + T − 1]. The data center decides real time purchase of power $E_{rt}(\tau)$ from the local grid, charging $R(\tau)$ and discharging $D(\tau)$ of batteries of the UPS to minimize the following problem

$$\min V\, E_{rt}(\tau)p_{rt}(\tau) + r(\tau)p_s(\tau) + X(t)\ [R(\tau)\eta - \mathrm{D}(\tau)/\eta]$$
$$s.t.\ (3)(6)(7)(8)$$

3. *Queue update*: Update the queues using Eqs. (5) and (15).

6 Performance Analysis

In this section, we analyze the performance bound of the Smart Data center algorithm.

Theorem (Performance Bound) The time-averaged cost ηRmax achieved by the Smart Data center algorithm based on accurate knowledge of $X(\tau)$ in the future coarse-grained interval satisfies the following bound with any nonnegative value of decision parameter V:

1. The time-average cost $\mathit{Cost\ of\ data\ center}_{avg}$ achieved by the algorithm satisfies the following bound:

$$\begin{aligned}\mathit{Cost\ of\ data\ center}_{avg} &\cong \lim_{t\to\infty} 1/t \sum_{\tau=0}^{t+T-1} \mathbb{E}\left[\mathit{Cost\ of\ data\ center}_{opertiaonal}(\tau)\right] \\ &\le \text{Ø}^{opt} + \left[\left\{\frac{1}{2} max\left[\eta^2 R^2\ (\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]\right\} \right. \\ &\left. + \frac{T(T-1)\left[\eta^2 R^2\ (\mathrm{t}),\ \frac{D^2(\tau)}{\eta_2}\right]}{2}\right]/V\end{aligned}$$

Where, Ø^{opt} is an optimal solution

Proof: Let $t = kT$ for nonnegative k and $\tau \in [t, t + T - 1]$. We first look at the optimal solution. In optimal solution, all the future statistics including power demand, surplus energy from the grid and energy prices are kown to the data center in advance. Due to knowledge of future, the data center can manage to reduce the real time purchase to zero. We can say the optimal solution is

$$\begin{aligned}\text{Ø}^{opt} &\cong \min\{E_{lt}(t)\, p_{lt}(t) + r(\tau)p_s(\tau) + \partial(t) * Cr\} \\ &s.t.\ \ E_{lt}\ (t) + D(\tau) + r(\tau) - C(\tau) = d(\tau), \\ &0 \le E_{lt}\ (t) \le Pgrid, \\ &\forall t\text{: } \mathit{constraints}\ (6)\ (7)\ (10).\end{aligned}$$

By using the optimal solution in right hand side of the Eq. (23), we get

$$\Delta LD_T(t) + V\mathbb{E}\left\{\sum_{\tau=t}^{t+T-1} Cost\ of\ data\ center_{opertiaonal}(\tau)|X(t)\right\}$$
$$\leq \left\{\frac{1}{2}max\left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]\right\}$$
$$+\frac{T(T-1)\left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]}{2} + V\text{Ø}^{opt}$$

Taking the expectation of the both sides and rearranging terms we get

$$\mathbb{E}\{L(t+T) - L(t)\} + VT\mathbb{E}\left\{\sum_{\tau=t}^{t+T-1} Cost\ of\ data\ center_{opertiaonal}(\tau)|X(t)\right\}$$
$$\leq \left\{\left\{\frac{1}{2}max\left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]\right\}\right.$$
$$\left.+\frac{T(T-1)\left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]}{2}+\right\}T + VT\text{Ø}^{opt}.$$

By taking the sum over $t = kT$, $k = 0,1,2,\ldots, k-1$ and dividing both sides by *VKT*, we get

$$\frac{1}{kT}\mathbb{E}\left\{\sum_{\tau=0}^{kT-1} Cost\ of\ data\ center_{opertiaonal}(\tau)\right\}$$
$$\leq \frac{\left\{\left\{\frac{1}{2}max\left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]\right\} + \frac{T(T-1)\left[\eta^2 R^2(\mathrm{t}),\ \frac{D^2(\tau)}{\eta^2}\right]}{2}+\right\}}{V} + \text{Ø}^{opt}.$$

As the variable k approaches to infinity, $k \to \infty$, the theorem is proved.

2. The UPS battery level $E_u(t)$ is bounded in the range $[E_u min,\ E_u max]$. There is always power remained in the batteries for backup in case of black out.

   Proof: We first prove that

$$-\frac{VPmax}{T} - \frac{Dmax}{\eta} \leq X(t) \leq E_u\mathrm{max} - \frac{VPmax}{T} - E_u\mathrm{min} - \frac{Dmax}{\eta}$$

We prove this by induction. For t = 0 we have

$$X(0) = E_u(0) - \frac{VPmax}{T} - E_u\mathrm{min} - \frac{Dmax}{\eta}$$

and $E_u\text{min} \leq E_u(0) \leq E_u max$. So we get

$$-\frac{VPmax}{T} - \frac{Dmax}{\eta} \leq X(0) \leq E_u\text{max} - \frac{VPmax}{T} - E_u\text{min} - \frac{Dmax}{\eta}$$

Now we consider $0 \leq X(t) \leq E_u\text{max} - \frac{VPmax}{T} - E_u\text{min} - \frac{Dmax}{\eta}$, therefore, there is no battery recharging, i.e., $R(t) = 0$. The maximum amount of power that can be discharged each time is $\frac{Dmax}{\eta}$.

Now we have

$$-\frac{VPmax}{T} - \frac{Dmax}{\eta} < -\frac{Dmax}{\eta} < X(t+1) \leq X(t)$$
$$\leq E_u\text{max} - \frac{VPmax}{T} - E_u\text{min} - \frac{Dmax}{\eta}.$$

For the case when $-\frac{VPmax}{T} < X(t) \leq 0$, $D(t) = 0$. The amount of power that can be charged and discharged at maximum each time are ηRmax and $\frac{Dmax}{\eta}$, respectively. We get

$$-\frac{VPmax}{T} - \frac{Dmax}{\eta} < X(t+1) \leq X(t) + \eta Rmax$$
$$\leq E_u\text{max} - \frac{VPmax}{T} - E_u\text{min} - \frac{Dmax}{\eta}.$$

Finally consider the case, when $-\frac{VPmax}{T} - \frac{Dmax}{\eta} \leq X(t) \leq -\frac{VPmax}{T}$ again D(t) = 0 as $X(t) \leq -\frac{VPmax}{T}$. We get

$$-\frac{VPmax}{T} - \frac{Dmax}{\eta} < X(t) \leq X(t+1) \leq E_u\text{max} - \frac{VPmax}{T} - E_u\text{min} - \frac{Dmax}{\eta}.$$

Using Equ. 14, we have

$$-\frac{VPmax}{T} - \frac{Dmax}{\eta} \leq X(t) = E_u(t) - \frac{VP_{max}}{T}$$
$$-E_u\text{min} - \frac{Dmax}{\eta} \leq E_u\text{max} - \frac{VPmax}{T} - E_u\text{min} - \frac{Dmax}{\eta}.$$

From all the cases, we can conclude that

$$E_u\, min < E_u(\tau) < E_u\, max.$$

3. All decisions are feasible.

   The smart data center algorithm makes decision to satisfy all the constraints. Therefore, the Smart Data center algorithm is feasible.

7 Related Work

The past decade has witnessed the enormous growth in the online applications and services. The online applications and services are hosted in data centers. With the increase demand of online services, the cost of power consumption in the data centers is increasing significantly. There is extensive existing research on the power management of data centers [1, 13, 22]. Most of the works focus on the reducing the power consumption in the data center using different schemes like voltage scaling, frequency scaling, and dynamic shutdown. However, the earlier works have not focused on reducing the overall cost of the power used in the data center.

Recently, the researchers have started to focus on reducing the cost of power utilized in the data center. *Ref.* [2, 20, 21], focused on migration of workload between different data centers to utilize the low electricity prices in different geographical locations. However, the emphasis is not on reducing the cost of a single data center.

For reducing the cost of a single data center, the researchers have emphasis on the power storage in the data centers. In [7–9, 23] and [24], the researchers have shown the importance of using UPS in the data center for reducing the overall cost of electricity in a single data center. However, the aforementioned works have not considered the multiple price markets to power up the data center.

In [10–12, 15] the authors have worked on energy procurement from long term, intermediate, real time markets. However, the approaches in the aforementioned schemes depends upon the forecasting techniques, such as dynamic programming and Markov decisions to know the power demand in advance. Similar to our work, Deng et al. [3, 4] have used two timescale Lyapunov optimization technique to reduce the cost of a single data center. They have utilized the long term and real time price market of a smart grid along with the On-Site wind or solar green energy. However, they have ignored the cost of On-Site wind or solar energy. We have considered the scenario of providing the power to the data center from multiple power grids, local grid for long term and real time market, whereas remote grid for low cost surplus energy.

8 Conclusions

In this work, we have targeted the key problem that how to minimize the cost of power consumption in the data center? We proposed the new idea to power up the data center from more than one Smart grid. We exploited the long term and real time price market from the local grid and low cost surplus power from the remote grid. We developed the algorithm titled "Smart Data center" that decide how much power to be purchased from the long term and real time market. We also utilized the Uninterrupted Power Supply (UPS) as back up in the data center. The performance of the "Smart Data center" algorithm is analyzed using theoretical analysis. The performance analysis of the algorithm using real world traces are left for future work.

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