

# Three-Dimensional agent-based model of fish collective behaviour using topological interaction

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**Abstract**—This paper presents a novel 3D agent-based model of fish collective behaviour based on topological interaction. To the best of our knowledge, it is the first 3D model that implements topological interaction. Despite the fact that the topological interaction was proposed based on the observation of collective behaviour of bird flock [1], simulations show that our model also displays realistic collective patterns of fish school as seen in nature. We also observe the automatic transition of different collective patterns, which is a natural phenomenon but failed to show in many previous agent-based fish school models.

**Keywords**—agent-based modelling; fish school; collective behaviour; swarm; topological interaction

## I. INTRODUCTION

From bird flock to fish school, tens of thousands animals coordinate as a cohesive individual. Such fascinating phenomenon of animal collective behaviour has attracted more and more attention in recent years. By using agent-based models, researchers have unveiled a plausible mechanism behind animal collective behaviour, self-organisation, that is, the global complex pattern of animal swarm emerges from local interacting behavioural rules of all individuals.

In order to understand the self-organizing behavioural rules that govern animal collective behaviour, in the past decade, researchers have proposed behavioural rules based on observations. Inada etc. proposed behavioural rules that define zones of attraction, repulsion and parallel-orientation based on metric (Euclidean) distances to investigate the order and flexibility in the motion of fish school [2]. The model was implemented in 2D. In [3], Couzin etc. proposed a 3D model of behavioural rules that are similar to Indana's rules to simulate spatial dynamics of fish school without predators. By adjusting the parameter of the behavioural rules, the 3D model exhibited four collective patterns (swarm, torus, dynamic parallel and highly parallel). The above self-organizing behavioural rules

are called metric interaction since the rules are based on metric distance between the agent and their neighbours.

In 2008, Ballerini etc. found that flock of starlings exhibits much higher cohesion than the model based on metric distance, which leads to a novel explanation to such high cohesion: topological interaction [1]. In their proposed behavioural rules, the interactions of individuals in bird flock are based on topological distance rather than metric distance, that is, the behaviour of an individual depends on the distance rank of the neighbour. The authors had also built a 2D model based on their topological behavioural rules to test their hypothesis.

In this paper, for the first time, we build a 3D agent-based model that implements the topological behavioural rules to simulate the spatial dynamics of fish school. From our model, we observe realistic collective patterns as observed in nature. We also observe the transition of the collective patterns automatically, which has been frequently observed in nature. However, the realisation of the transition of different collective patterns has not been reported in the literature of agent-based modelling, except for the 1D model presented in [4], which displayed automatic transition between swarm and dynamical parallel.

The paper is organised as follows. In Section II, we briefly introduce our model, especially the topological interaction. We will give the simulation results of the proposed model in Section III. We will conclude this paper in Section IV.

## II. THE TOPOLOGICAL INTERACTION MODEL

In our topological interaction model, an individual interacts with neighbours of a fixed number to determine its velocity and position, including attraction among the individuals, short range repulsion, and alignment of the velocities [2].

Given a number ( $TN$ ) of individual fishes moving in a continuous 3D space, the driving rules on each fish  $j$  are

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This work is funded in part by National Natural Science Foundation of China (grants No. 61272314), the Program for New Century Excellent Talents in University (NCET-11-0722), the Fundamental Research Funds for the Central Universities (CUG, Wuhan), the Specialized Research Fund for the Doctoral Program of Higher Education (grant No. 20110145110010), the Programme of High-Resolution Earth Observing System (China), and the Hundred University Talent of Creative Research Excellence Programme (Hebei, China).

illustrated as in Figure 1. The model assumes a discrete time, i.e., the model evolves in a sequence of identical timesteps (denoted by  $\tau$ ) corresponding to fish's response latency. The velocity and position of  $j$  at time  $t$  are written as  $\mathbf{V}_j(t)$  and  $\mathbf{P}_j(t)$  respectively. The expectation of  $j$ 's velocity at the successive time step ( $t+\tau$ ) is denoted by  $\mathbf{d}_j(t+\tau)$ , which is an ideal result

from  $j$ 's observations on its neighbors within its sight. In reality, a fish may not sense all of its neighbors, and the model assumes a blind volume for each fish which is shown as the inner space of the cone with a point angle of  $2\alpha$  right behind  $j$ . The collective influences of neighbors on  $j$  vary in their distances to  $j$  in descending order.

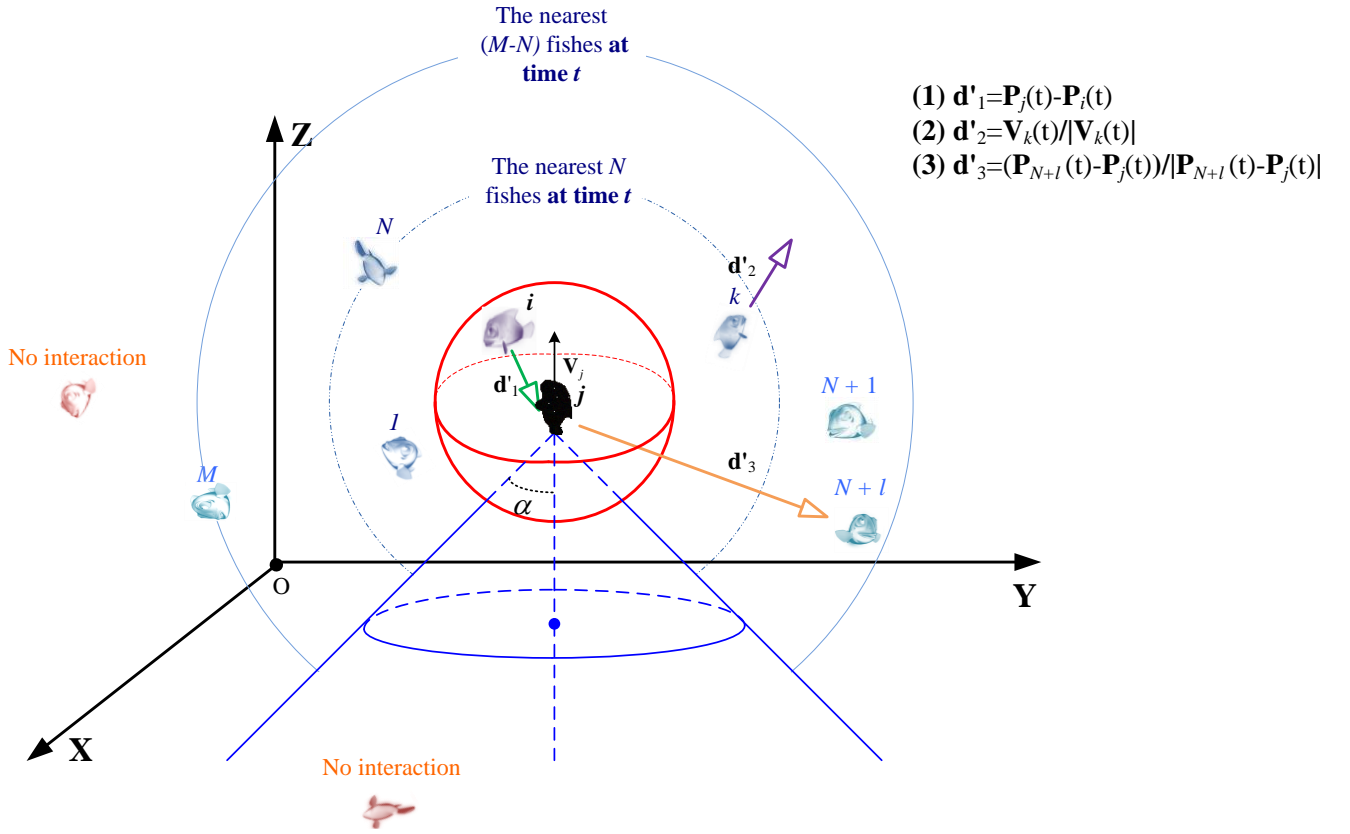


Figure 1. Topological distance interaction model

Rule 1 (repulsion): Each individual tends to maintain a minimum distance (fixed to  $r$ ) from others. If an individual  $j$  senses another individual  $i$  present within this distance at time  $t$ , it moves away from  $i$ .

$$\mathbf{d}_j^1(t+\tau) = \mathbf{P}_j(t) - \mathbf{P}_i(t) \quad (1)$$

where  $\mathbf{P}_i(t)$  is the position of individual  $i$  at time  $t$ , apparently, here we have  $|\mathbf{P}_j(t) - \mathbf{P}_i(t)| < r$ . Individual  $j$  responds in this manner only under the influence of  $i$  with rest individuals ignored, i.e.,  $\mathbf{d}_j(t+\tau) = \mathbf{d}_j^1(t+\tau)$ .

When  $j$  senses no neighbor within  $r$ , the expectation of its velocity is subject to the  $M$  nearest neighbors. Let's sort these  $M$  individuals in ascending order according to their distances to  $j$ , individual  $j$  responds to the first  $N$  neighbors with a tendency to align itself with the  $N$  nearest neighbors' velocities; it responds to the rest  $(M-N)$  nearest neighbors by being attracted to them. Apparently, the ranges of the two groups of nearest neighbours may vary from time to time.

Rule 2 (alignment): For any individual  $k$  amongst the  $N$  nearest individuals,  $j$  responds to  $k$ 's presence with a tendency

to align with  $k$ 's velocity. The collective influences on  $j$  of the  $N$  individuals conform to formula 2.

$$\mathbf{d}_j^2(t+\tau) = \sum_{k=1}^N \frac{\mathbf{V}_k(t)}{|\mathbf{V}_k(t)|} \quad (2)$$

where  $\mathbf{V}_k(t)$  is the velocity of individual  $k$  at time  $t$ .

Rule 3 (attraction): For the rest  $(M-N)$  nearest individuals, individual  $j$  is attracted to them which moving towards them. The collective influences on  $j$  of the  $(M-N)$  individuals conform to formula 3.

$$\mathbf{d}_j^3(t+\tau) = \sum_{l=1}^{M-N} \frac{\mathbf{P}_{N+l}(t) - \mathbf{P}_j(t)}{|\mathbf{P}_{N+l}(t) - \mathbf{P}_j(t)|} \quad (3)$$

where  $\mathbf{P}_{N+l}(t)$  is the position of individual  $(N+l)$  at time  $t$ .

From rules 2 and 3, when no individual is present within a distance  $r$  to  $j$ , the expectation of  $j$ 's velocity at the next time step is  $\mathbf{d}_j(t+\tau) = \mathbf{d}_j^2(t+\tau) + \mathbf{d}_j^3(t+\tau)$ .

Let's write the included angle between  $\mathbf{d}_j(t+\tau)$  and  $\mathbf{V}_j(t)$  as  $\beta(t)$  and define a preset threshold of the turning angle of an individual as  $\theta$ . If  $\beta(t) < \theta$ , the actual velocity of  $j$  at the next time step is:

$$\mathbf{V}_j(t+\tau) = \frac{\mathbf{d}_j(t+\tau)}{|\mathbf{d}_j(t+\tau)|} \times s \quad (4)$$

where  $s$  is a constant representing the speed of an individual.

If  $\beta(t) \geq \theta$ , vector  $\mathbf{V}_j(t)$  turns towards  $\mathbf{d}_j(t+\tau)$  in an angle of  $(\theta+\varepsilon)$  and results in a new vector  $\mathbf{d}'_j(t+\tau)$ , where  $\varepsilon$  is a random angle with a value in  $[-\theta/2, \theta/2]$ . The actual velocity of  $j$  at the next time step is:

$$\mathbf{V}_j(t+\tau) = \frac{\mathbf{d}'_j(t+\tau)}{|\mathbf{d}'_j(t+\tau)|} \times s \quad (5)$$

Individual  $j$ 's position at the next time step is:

$$\mathbf{P}_j(t+\tau) = \mathbf{V}_j(t+\tau) \times \tau + \Delta\mathbf{P} \quad (6)$$

where  $\Delta\mathbf{P}$  is defined to offset the potential overlap between any two individuals (see Fig. 2). Assume the length of an individual is  $len$ , when  $len$  is larger than the distance from  $j$  to another individual,  $\Delta\mathbf{P}$  can be calculated as formula (7); otherwise  $\Delta\mathbf{P}$  is set to  $\mathbf{0}$ :

$$\Delta\mathbf{P} = \sum_{i=1, i \neq j}^{TN} (len - |\mathbf{P}_i - \mathbf{P}_j|) \times \frac{\mathbf{P}_i - \mathbf{P}_j}{|\mathbf{P}_i - \mathbf{P}_j|} \quad (7)$$

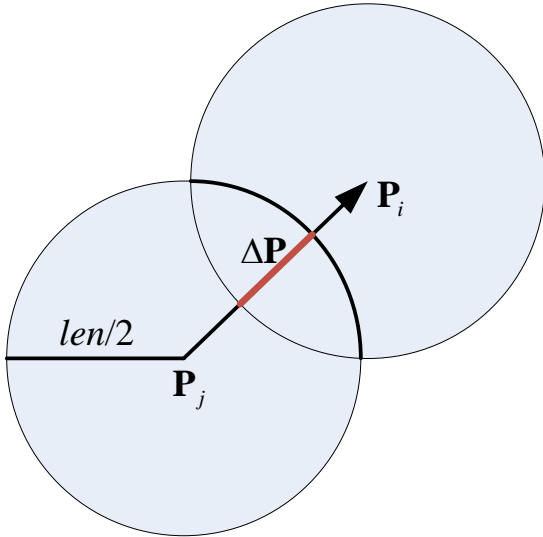


Figure 2. Illustration of offsetting the potential overlaps between fishes

### III. RESULTS

We have performed a number of simulations to examine the proposed model using topological interaction. The parameter settings are presented in Table 1. During the simulations, the simulated fish school autonomously exhibits collective behaviors of swam, torus, and dynamic parallel (see Fig. 3).

Automatic switch amongst the three behavioral patterns can also be observed during the simulations.

We calculated the average speed of the whole fish school at time  $t$ , written as  $P_{group}(t)$ :

$$P_{group}(t) = \frac{1}{TN} \times \left| \sum_{j=1}^{TN} \frac{\mathbf{V}_j(t)}{|\mathbf{V}_j(t)|} \right| \quad (8)$$

As shown in Fig. 4, when the fish school exhibits torus behavior,  $P_{group}(t)$  has a very small value in between  $[0, 0.15]$ ; when in swarm status, the fish school's  $P_{group}(t)$  has a value in between  $(0.15, 0.6]$ ; when in dynamic parallel status,  $P_{group}(t)$  increases to be in between  $(0.6, 1.0]$ . From the curve, we can see the switches between the behaviors, and the duration of the torus behavior is the longest.

TABLE I. PARAMETER SETTING OF THE SIMULATIONS

Parameters	Values
Number of individuals ( $TN$ )	200
Average length of individuals ( $len$ )	8
Distance for repulsion behavior ( $r$ )	10
Number of nearest neighbors ( $M$ )	10
Number of neighbors triggering alignment behavior ( $N$ )	5
Number of neighbors triggering attraction behavior ( $M-N$ )	5
Timestep ( $\tau$ )	0.5
Threshold of turning angle ( $\theta$ )	$10^\circ$
Random variation of individual's turning angle ( $\varepsilon$ )	$-5^\circ - 5^\circ$
Point angle of the blind volume ( $2\alpha$ )	$140^\circ$
Individual speed ( $s$ )	5

### IV. CONCLUSIONS

In this study, we built a 3D agent-based model to simulate the spatial dynamics of fish school. Significantly different from previous work in the literature, our model, for the first time, implemented the topological interaction behavioural rules, which was originally proposed to simulate bird flock, for the simulation of fish school in 3D. The swarm patterns emerged from the 3D model are very similar to those observed in nature. From the simulation of our model, we also observed the automatic transition of different swarm patterns as seen in nature, which has not been reported in any previous work. Our model suggested the plausibility of topological interaction rules to simulate and to understand a variety of animal collective behaviours.

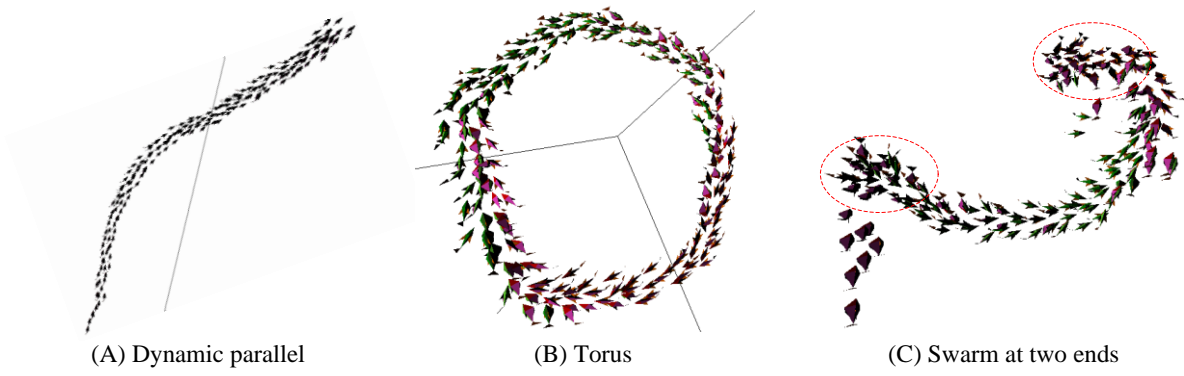


Figure 3. Illustrating the fish school's swarm patterns

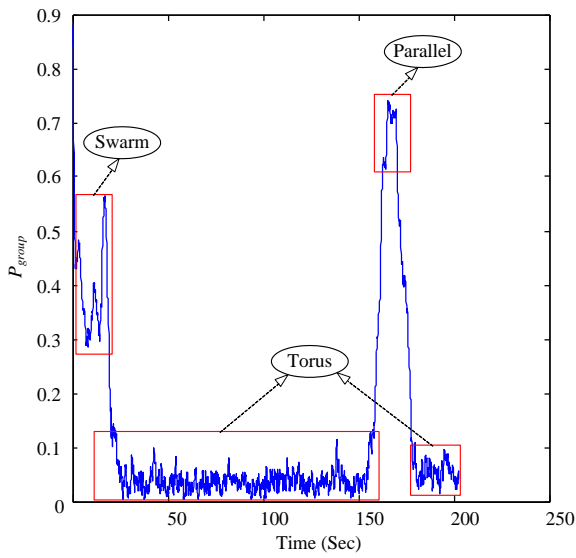


Figure 4. Evolution of the group average velocity  $P_{group}$

#### ACKNOWLEDGMENT

Dr. Shan He would like to thank the financial support of the Systems Science for Health initiative of the University of Birmingham.

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